What is basic physics worth?

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Orders of magnitude, energy, and overconfidence in technical refinements

François Roby*

« Être informé de tout et condamné ainsi à ne rien comprendre, tel est le sort des imbéciles. »

Georges Bernanos (1888-1948), in *La France contre les robots*

"It doesn't make any difference how beautiful your guess is,

it doesn't make any difference how smart you are, who made the guess, or what his name is.

If it disagrees with experiment, it's wrong. That's all there is to it."

Richard Phillips Feynman (1918-1988), in a famous 1964 lecture at Cornell University.

Physics is often perceived as a science of complex and precise calculations, making possible any sorts of technical "miracles" in the midst of which we live. However, the basis of the discipline does not lie in these refinements, but in a small number of laws that should be rigorously applied; it also lies in the physicists' ability to perform justified approximations.

A striking and well documented feature of 9/11 attacks in New York City is persisting fires and high temperatures in the World Trade Center ruins. Infrared thermography measurements have been made, just after the event as well as weeks and months later, which allow to estimate surface temperature and corresponding areas, and the cooling characteristic time of the place.

Cooling of a hot body in a colder environment occurs thanks to conduction, convection and radiation. Taking into account only free convection, and performing only orders of magnitude calculations because of a lack of accurate data, it is possible to get a lower estimate of the total heat released at Ground Zero.

Combining the heat estimate with the maximum energy/mass ratio of all chemical carriers, we can rule out these as the source of heat released at Ground Zero and therefore consider nuclear energy as the only available solution.

Only the opportunistic use of a built-in nuclear demolition feature, designed at the same time as the World Trade Center itself, is a technically viable explanation. Some literature search about pacific use of nuclear explosives as envisaged in the 1960s shows that such an idea, if surprising today, was not unthinkable in the context of the time.

Using such sources we propose a general mechanism for the destruction of the World Trade Center, focusing on the shockwave ef-

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fects and on subsidence crater formation. Some secondary effects need further investigation.

1 Introduction

It is usually believed that basic physics, such as classical mechanics, electromagnetism, optics or any other field that students learn at undergraduate levels, is a necessary step towards more elaborate physics specialties but can never by itself lead to striking discoveries at the fringe of scientific knowledge, since it addresses only well-established concepts that have been used for decades or even, quite often, for centuries. It is true that no one will ever be able to "discover" that, for instance, Newton's laws of motion are false, since it is already known that they *are* indeed false or, to write it more precisely, that their domain of validity is limited and does not extend as far as, for instance, high energy particle physics.

Although we do not challenge this obvious fact, we will show in this paper that much more than a mere "necessary step" is to be expected from basic physics, especially at a time when computer simulations, although being extremely valuable tools for solving complex problems - particularly in engineering areas - sometimes lead to a "black box" thinking that obscures simple and powerful physics concepts. Because computer simulations have become "too easy a method" for solving even simple physics problems - some scientists endangering further the understanding when talking about "computer *experiments*" instead of "computer simulations" - it is of great interest to call back "good old methods" of physics, those of the pre-computer era when experiments were only genuine ones and basic understanding was required before performing them, or before performing tedious analytical calculations, which can also sometimes muddle up understanding by diverting too much of a scientist's effort in solving equations instead of concentrating on the underlying concepts.

A practical example will be given through a well-known, yet poorly-understood - even among the scientific community - energy problem, emphasizing the need to *limit* the modelling level of complexity instead of trying to make the model as close to reality as possible, as it is generally the rule when working with computer simulations. If the aim is to rigorously understand what is really going on at a fundamental level, it is important to avoid elaborating hypothesis which are nothing but mere speculations, and this can be done only when working on simple, clear ideas.

Our purpose is *not* to denigrate the use of computer simulations, which have proven to be effective, fast and often irreplaceable tools for physics and engineering, but to show that they should not be used in the first place when a direct, human-made argument gives an answer to the problem - although a simplified one - and leads to a deeper, yet easier to share among "ordinary humans", understanding. Since science is not only valuable for its technological applications but also for its educative value, such a perspective should not be considered, according to us, as an old-fashioned or a limited-budget way of doing physics, but as the primary and most important one before any technological refinement is called on for help. And especially when dealing about complex problems where risk of error is high: *"safety first"*, as sensible sailors or alpinists would say.

2 Orders of magnitude: the Fermi approach

2.1 The classical piano tuners problem

A story often narrated to students for educative purposes is how Enrico Fermi, the famous Italian physicist and 1938 Nobel Prize winner, used to ask his students to find answers - although approximate - to almost all questions, including the ones which have little or no link to physics, using simple logic and dimensional analysis [1]. The most popular example of this, the “classical Fermi problem”, was to guess the number of piano tuners in Chicago. Independantly of the numerical data, such an answer can be found using the following reasoning:

- A piano needs to be tuned from time to time, let's say n times a year.
- The operation takes some time to be performed (including travel time), let's call it Δt.
- A piano tuner works like other workers, a finite amount of time per year, let's say wh_y hours a year.
- Not all households have a piano; moreover, not all households have a piano that is tuned regularly. Let's call f the fraction of households that have got one regularly tuned piano (the number of households having more than one regularly tuned piano will be considered negligible).
- There is an average of p persons per household.
- There are C inhabitants in Chicago.

When all these parameters are known, then the answer (let us call N_{pt} the number of piano tuners) can be computed as follows:

- There are $\frac{C}{p}$ households in Chicago and $f\frac{C}{p}$ pianos that need to be tuned regularly.
- There are $nf\frac{C}{p}$ tunings that are made per year in Chicago, and they necessitate $nf\frac{C}{p}\Delta t$ working hours.
- To perform this work a number of $N_{pt} = nf\frac{C}{p}\frac{\Delta t}{wh_y}$ full-time piano tuners is needed.

Of course any numerical answer will depend on the quality of the numerous estimates made; however, each of them should be easily performed by anyone if only the right order of magnitude is sought for. And furthermore, there is a reasonable chance that errors in different estimates will more or less compensate. Let's give a numerical illustration:

- $n = \frac{1}{year}$
- $\Delta t = 2\,hours$
- $wh_y = \left(40\,\frac{hours}{week}\right)\left(50\,\frac{weeks}{year}\right) = 2000\,\frac{hours}{year}$
- $f = \frac{1}{20}$
- $p = 2$
- $C = 3.5 \times 10^6$ (1940 value, urban area not taken into account)

With these numbers the answer is:

$$N_{pt} = \frac{1}{year}\frac{1}{20}\frac{3.5 \times 10^6}{2}\frac{2\,hours}{2000\frac{hours}{year}} = 87.5 \tag{1}$$

Of course such a non-integer number is absurd and must be rounded up to 90, or even 100 since one should not expect better than a crude estimate of the real number; but the real point is that it can't be only one, nor 10^4.

2.2 The pinhole camera problem

Dimensional analysis is an important part of the game, although the classical Fermi problem deals more with numerical estimates and straightforward thinking than with checking units. Let us take a slightly different example to illustrate this, where actually no numerical guess has to be performed but where the right answer comes *only* from dimensional analysis.

A pinhole camera is the most primitive type of camera [2], consisting only in a small hole punctured on one side of a light-proof box. It produces, just as an ordinary camera, inverted real images of the surroundings on the side of the box facing the pinhole - let us call this side the image plane. There is an optimal size for this hole: if it is too large, light rays coming from an object point outside of the box will be able to strike the image plane within a rather large image spot, only because of straight propagation of light according to geometrical optics, and this will cause image blur. Conversely, if the hole is too small geometrical optics is not valid any more, diffraction occurs and enlarges the image spot also. A rigorous calculation of the optimum pinhole diameter can be performed using wave optics; Josef Petzval has proposed one in the mid-19th century and gave the result[3]:

$$d = \sqrt{2f\lambda} \tag{2}$$

where λ is the wavelength of the light and f the distance between the pinhole and the image plane - equivalent to the focal length in a camera with lens. However this result, excepted the $\sqrt{2}$ dimensionless factor that is rather close to unity, can be inferred very quickly using only dimensional analysis. The answer must depend on the wavelength λ, because diffraction is involved; it must also depend obviously on the distance f between the pinhole and the image plane for equally obvious geometrical reasons. These are the only two parameters of the problem, and one must get from them an optimum diameter d which is also a length. To get a length from two other lengths could be done mathematically using a sum or a difference, but this would have no physical meaning since λ being much smaller than f, a sum or a difference would practically keep only the largest quantity. Hence, the most straightforward mathematical formula for getting the optimal diameter is $d = \sqrt{f\lambda}$, which is almost the right one and gives at least the right order of magnitude, since any dimensionless factor like $\sqrt{2}$ here can not alter the order of magnitude.

3 A practical example: disproving some "conspiracy theories"

The terrorists attacks that occured in several places of the USA on September 11, 2001 have been since the event the subject of many controversies, for instance regarding the very fact that a plane really strucked the Pentagone or a missile instead [4]. Most mass media, and even some scientists [5], denigrate people who look for explanations of well-known events that depart from the one provided by officials, calling them "conspiracy theorists", although in the case of September 11, 2001 attacks, the widely accepted version is undoubtely also of a conspiracy type. Nobody claims these horrific events were only accidents, and everyone acknowledges they were planned in advance by some criminal individuals which is the very definition of a conspiracy. It would be more correct in this case, therefore, to call them "alternative conspiracy theorists". As physics does not deal with human intentions, it cannot address directly the "conspiracy" item - which is anyway, as we pointed out, irrelevant here. But as it deals with natural laws, it can refute some explanations which do not fulfill the necessary requirement of being compatible with these laws, just as a crime laboratory is able to rule out some murder suspects.

Most "conspiracy theories" about September 11, 2001 events - including the widely accepted one, since 4 passenger airliners being hijacked by 19 terrorists is certainly the result of a high-level kind of conspiracy - deal with complex phenomena like collision between airplanes and buildings or the catastrophic collapse of skyscrapers. Surprisingly, people who discuss these issues - for instance, arguing about what made three skyscrapers collapse in a few seconds - elaborate from the very beginning some complex scenarii without even checking the most obvious and well-tried laws of physics, such as conservation of energy. As we will see, such an approach turns out to be ineffective and to obscure even more an already difficult problem. Complex and questionable arguments should always be used to refine the conclusions of straightforward and robust ones, not the other way round.

Some authors [6] have challenged in 2013, using a detailed analysis, a series of papers by Bažant *et al.* who pretended to explain rationally the dominant narrative[1] regarding the World Trade Center skyscrapers collapses, which attributes them to fires weakening their structure. More recently (2016), others [7] using only simple mechanics have successfully shown this narrative to be incompatible with physics laws.

We shall here try to go one step further and, without using mechanics at all but rather thermodynamics in its simplest form, *exclude* from the range of options some explanations that have been advanced for the buildings collapses. For this we will rely almost exclusively on the basic concepts of thermodynamics, being taught at an entry-level course of physics: namely, the first law which states that energy is conservative and that work and heat are two kinds of energy that can transform in each other, and the second law which states that heat can only flow *spontaneously* from warmer bodies to colder ones, according to Clausius statement. And to make the argument easier to expose and - above all - to understand by any reader, we will adopt in the following a kind of "Fermi approach" and ignore unnecessary details to focus on a simple energetics problem, for which experimental data need only to be known at the order of magnitude level.

[1] We use here the adjective "dominant" in the sense where it is massively reported by mass media and governments, not in the sense that it should be considered as more plausible or more correct on a scientific basis.

3.1 Aftermath of 9/11 terrorist attacks: Ground Zero persistent high temperatures

It is a widely documented fact that persistent fires occurred at Ground Zero in Manhattan as an aftermath of the September 11, 2001 terrorist attacks. Numerous newspapers as well as broadcasted news bulletins narrated the extremely long effort made by firefighters to secure the place, and it was reported that "*The underground fire burned for exactly 100 days and was finally declared "extinguished" on Dec. 19, 2001.*" [8]. We will give here only a few examples of testimonies related to these very unusually high and long-lasting temperatures:

- James M. Williams, then president of the Structural Engineers Association of Utah, wrote in the October 2001 issue of the newsletter of his association (p.3): "*As of 21 days after the attack, the fires were still burning and molten steel was still running.*" [9] Note that the melting point of steel, which depends on its chemical composition, is close to 1700 K.
- James Glanz wrote in the *New York Times* on November 29, 2001, about the "*strange collapse of 7 World Trade Center*", citing Dr. Barnett, a professor of fire protection engineering at the Worcester Polytechnic Institute: "*A combination of an uncontrolled fire and the structural damage might have been able to bring the building down, some engineers said. But that would not explain steel members in the debris pile that appear to have been partly evaporated in extraordinarily high temperatures*". [10] Note that the boiling temperature of iron, the main component of steel, is 3134 K.
- William Langewiesche, the only journalist to have had unrestricted access to Ground Zero during the cleanup operation, states in the book "*American*

Ground: Unbuilding the World Trade Center" the following (pp. 31-32): "*He would go wandering off through the subterranean ruins, [...] with apparently only a vague awareness of the danger signs around him — the jolt of a collapse far below, [...] or, in the early days, the streams of molten metal that leaked from the hot cores and flowed down broken walls inside the foundation hole.*" [11]

- during the public hearing for the National Commission on Terrorist Attacks Upon the United States, on April 1, 2003, Ken Holden (New York Department of Design and Construction) stated: "*Underground, it was still so hot that molten metal dripped down the sides of the wall from Building 6.*" [12]
- New York firefighters have recalled "*heat so intense they encountered rivers of molten steel.*" [13]

Famous photographs of red-glowing steel removed from the pile were published, and some interviews of firefighters were made where it was claimed that flows of liquid metal were flowing underneath the debris, "like in a steel plant". Note that there was also evidence (see for instance [7], Fig. 6) of glowing molten metal pouring out of WTC2 continuously for 7 minutes before its collapse, but we will *not* address this particular feature since our aim is only to understand the origin of persistant high temperatures in Ground Zero ruins, and not to document or study what happened before the collapse of the 3 buildings.

To summarize, ample evidence exists that can rule out fires as the heat source at Ground Zero in the weeks - or even months - following the attacks, since the extremely high temperatures encountered there would violate the second law of thermodynamics. Recalling that steel industry has only succeeded at the end of the nineteenth century to produce temperatures high enough to process steel in the molten state, it is clear that fires that were persisting for 100 days at Ground Zero were the *consequence*, not the *cause*, of an extremely large heat source which temperature was, during a long time, much higher than that of common building fires, be they collapsed or not.

3.2 Estimating energy released from Ground Zero: the Fermi approach

Giving a precise estimate of the amount of heat released at Ground Zero is an almost impossible task, since it would require a huge amount of experimental data (mainly temperature measurements, in a lot of places and repeated over the cooling down time which lasted for months) and since heat transfer in such a complex environment as the debris pile involves several mechanisms (conduction, convection - both free and forced, and radiation) that can only be accurately calculated using numerical computation based on these experimental data.

Furthermore, huge amounts of water percolated through the debris, contributing to the cooling process by elevating the temperature of water which was drained away, but also for some part by evaporation, as white plumes on the site demonstrate: fires are very unlikely to produce white plumes, especially in an oxygen-starved environment like underground remnants. It is very difficult to estimate the cooling contribution of this water, since it would require the knowledge of both the volumes and the temperature differences; an article submitted to the 23rd American Chemical Society National Meeting (Orlando, FL, April 7-11, 2002) stated [14] that for the first 10 days after the attacks, roughly 30 million gallons ($\approx 114.10^3\, m^3$) water percolated through the debris, based on the pumping records. From this volume roughly 1 million gallon fell on the site (the so-called "bathtub" area) because of rain, 3 million gallons were hosed in the fire-fighting efforts and consequently 26 million gallons, i.e. the main part, came from leaks in the "bathtub", which was proven to be seriously damaged.

However, if we intend only to give a rough estimate - at the order of magnitude level - of this heat, and furthermore, if we are satisfied with only a *lower limit* of this energy, then the work becomes much easier and we can use a kind of "Fermi approach" to get the result. As energy is the integral of power over time, and as any cooling process described by linear heat transfer equations in a fixed temperature environment leads to an exponential decay of temperature and thermal power (heat transfer rate), we only need to estimate the following:

- thermal power released at Ground Zero at some time, let us call it $P(t)$; this thermal power is proportional to the temperature difference ΔT with the environment if the cooling process can be described with linear equations;
- characteristic time of cooling process, let us call it τ.

Then the heat Q can be expressed integrating thermal power over time (here $t = 0$ represents September 11, 2001):

$$Q = \int_0^\infty P(t)\, dt \tag{3}$$

and assuming exponential decay of temperature difference ΔT, hence of thermal power, with a characteristic time τ, the calculation is straightforward:

$$Q = \int_0^\infty P_0\, e^{-\frac{t}{\tau}}\, dt = \tau P_0 \tag{4}$$

Again, as several cooling processes are at play with different characteristic times, this calculation should not be considered as an accurate one, but rather as a way to get a lower boundary for the real amount of heat released at Ground Zero - let us call it Q_{GZ} - since all contributions give positive values. One can only hope to get the right order of magnitude of Q_{GZ} if one chooses the dominant cooling process, which for such a problem (hot ground in contact with atmosphere for months) is known to be usually a free convection mechanism within the air. However, radiation might play an important and even dominant role at the beginning where surface temperatures were proved to be extremely hot, because of the T^4 dependance of Stefan-Boltzmann law. We provide in Appendix A a crude estimate of heat released by radiative transfer and show that its contribution should have been comparable to that of free convection.

Conduction in the ground is difficult to estimate but given the poor thermal conductivity of it is relatively minor, and anyway gives a positive contribution which we can neglect if we are satisfied with a lower estimate.

Forced convection because of water sprayed by the firefighters or leaked through the damaged "bathtub", if not negligible, is also restricted to the first weeks after the attacks and therefore should not be a dominant part of the cooling process, given the extremely long characteristic time of it as we will see later. Taking the order of magnitude cited above ($\sim 10^5$ m^3), the heat capacity of liquid water ($C_p = 4.18 \times 10^3$ J.kg^{-1}.K^{-1}) and assuming a maximum temperature difference $\Delta T = 50$ K between water "in" and water "out", the water could have taken away some $10^8 \times 4.18 \times 10^3 \times 50 \sim 2 \times 10^{13}$ J ~ 20 TJ of heat, which sounds huge but is still much less than the total amount of heat we will estimate in the following. Assuming all the water was transformed into vapor, and considering its heat of vaporisation of ≈ 2.26 MJ.kg^{-1}, this would translate in more than 250 TJ of heat which is much more but most probably over-estimated, although the persistence of white "fumes" at Ground Zero for weeks means that a large quantity of water did evaporate because of underground heat.

It can be argued also that ambient temperature was not constant at Ground Zero, both because of daily oscillations and because of weather variations, the cooling process having taken place during months; however, given the very important temperature difference between the place and the air (see below), this can not lead to a major change in the result. Moreover, since September 11 is at the end of summer in northern hemisphere, if we take as the ambient temperature value the one that New York experienced at this date (or, in a more relevant way, the mean value of the corresponding week), we underestimate the heat release rate for the cooler times of autumn and winter, which is consistent with our approach of giving a lower estimate of Q_{real}.

3.3 Heat transfer by free convection: the basics

We recall here what can be found in any heat transfer textbook; see for instance [15]. Free - or natural - convection is a phenomenon that occurs in a gravitational field because the volumic mass of a fluid varies with temperature, inducing buoyancy forces. Heat transferred by free convection is solely determined by these forces - hence by volumic mass differences - and by the fluid heat capacity and viscosity.

3.3.1 Preliminary remarks about ideal gases

In the case of gases, and especially the simple case of ideal gases, the variation of volumic mass with temperature does not depend on chemical peculiarities of the fluid but relies solely on the ideal gas law:

$$pV = nRT \tag{5}$$

where p is the pressure, V the volume, T the absolute temperature, n the number of moles of the gas, and $R = k_B \mathcal{N}_A \approx 8.314\,\mathrm{J.K^{-1}.mol^{-1}}$ the ideal gas constant, with k_B Boltzmann's constant and $\mathcal{N}_A$ Avogadro's number.

Using the molar mass M, the volumic mass of the gas is given by:

$$\rho = \frac{M}{V} = \frac{Mp}{RT} \tag{6}$$

and therefore varies proportionally with pressure and as the inverse of absolute temperature.

Furthermore, the heat capacity of ideal gases is also independent of the chemical nature of the gas, since it is a direct consequence of the equipartition theorem; any introductory course in thermodynamics demonstrate that molar heat capacities at constant volume, C_V, and constant pressure, C_p, depend directly on the number of degrees of freedom of the molecules. For instance, for diatomic molecules at room temperature, where rotational degrees of freedom must be taken into account but not vibrational ones, five degrees of freedom exist and this translates directly into the following results:

$$C_V = \frac{5}{2}R \tag{7}$$

$$C_p = C_V + R = \frac{7}{2}R \tag{8}$$

Let us recall here, just to put these results into an historical perspective, that the high temperature limit of molar heat capacity for solids was known to be roughly a constant as soon as 1819, thanks to French physicists Pierre Louis Dulong and Alexis Thérèse Petit, who expressed it in the so-called Dulong-Petit law (molar $C_p = 3R$), and even though the ideal gas constant had not been defined yet.

Viscosity of ideal gases can be inferred from the kinetic theory of gases and was shown experimentally to be independent of density by James Clerk Maxwell in a famous 1866 article [16]. It can be theoretically expressed as follows (see for instance [17]):

$$\eta = \frac{1}{3} N m \bar{v} l$$

where N is the number of molecules per unit volume, m their mass, $\bar{v}$ their mean velocity and $l = \frac{1}{\sigma N}$ their mean free path, where σ is the cross-sectional area of the gas molecules. As a consequence, the product Nl does not depend on N and the dynamic viscosity only depends on the mass of the molecules and the absolute temperature *via* the mean velocity $\bar{v} = \sqrt{\frac{3k_B T}{m}}$.

These preliminary remarks show that heat transfer by free convection in the air at standard pressure is governed by universal, well-known physics laws that do not depend on peculiarities of the problem. Therefore, heat transfer coefficients used in the following, although depending on precise geometry of the heated surfaces if one needs to know their precise values, will also obey universal laws which provide easily numerical values, at least at the order of magnitude level.

3.3.2 Similarity considerations

On a more experimental perspective, convection heat transfer - either free or forced - will be expressed through a convection heat transfer coefficient, usually written h, by the following equation:

$$\dot{q} = h\left(T_S - T_\infty\right) \tag{9}$$

where $\dot{q}$ is the convective heat flux (in $W.m^{-2}$), and T_S and T_∞ the surface and bulk fluid temperature, respectively. Consequently, h will be expressed in $W.m^{-2}.K^{-1}$.

If the underlying physical mechanism of free convection is simple, detailed analysis of a specific free convection case can be very complex due to intricacies of fluid dynamics for various geometries; analytical solutions are often not available. However, as soon as we deal only with orders of magnitude as it is the case in this paper, we only need to know how to classify the present problem among a limited number of typical cases and to apply some similarity considerations. We recall below the basics of these considerations, that are thoroughly developped, for instance, in Chapter 9 of [15].

We will only consider free convection flows bounded by a surface, as it is the case of interest for a hot surface in contact with atmosphere.

Let us first introduce the ratio of inertial to viscous forces acting on a fluid, through the dimensionless Reynolds number Re:

$$Re \equiv \frac{\text{inertial forces}}{\text{viscous forces}} = \frac{VL}{\nu} \tag{10}$$

where:

- V is the maximum velocity of the flow;
- L is a characteristic length of the problem;
- ν is the kinematic viscosity of the fluid, in $m^2.s^{-1}$, or ratio of the dynamic viscosity μ to the volumic mass ρ of the fluid: $\nu = \frac{\mu}{\rho}$.

Let us then define the ratio of buoyancy forces to viscous forces acting on a fluid, through the dimensionless Grashof number Gr:

$$Gr \equiv \frac{\text{buoyancy forces}}{\text{viscous forces}} = \frac{g\beta\left(T_s - T_\infty\right)L^3}{\nu^2} \tag{11}$$

where:

- g is gravitational acceleration (gravity of Earth);
- β is the volumetric thermal expansion coefficient of the fluid, defined as $\beta = -\frac{1}{\rho}\left(\frac{\partial\rho}{\partial T}\right)_p$ or as $\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p$, in K^{-1}. Note that in the simple case of an ideal gas, we have $pV = nRT$ and hence $\beta = \frac{1}{V}\frac{nR}{p} = \frac{1}{T}$.

Now let us define the ratio of viscous diffusion rate to thermal diffusion rate, through the dimensionless Prandtl number Pr:

$$Pr \equiv \frac{\text{viscous diffusion rate}}{\text{thermal diffusion rate}} = \frac{\nu}{\alpha} \tag{12}$$

where α is the thermal diffusivity, in $m^2.s^{-1}$. Note that the kinematic viscosity ν is also called the momentum diffusivity.

The convection heat transfer coefficient h, which is what we're looking for, will be obtained through the dimensionless Nusselt number, Nu:

$$Nu \equiv \frac{hL}{k} \quad (13)$$

where k is the thermal conductivity of the fluid, in $W.m^{-1}.K^{-1}$. In the general case Nu is a function of Re, Gr and Pr; however, forced convection effects may be neglected when $Gr/\left(Re^2\right) \gg 1$ and in this case, Nu is only a function of Gr and Pr.

3.3.3 Special case: upper surface of heated plate

Free convection being a gravity-driven phenomenon, orientation of the surface with respect to gravity acceleration vector, as well as relative temperature of the surface (hotter or cooler) with respect to the surrounding fluid, are the main parameters used to classify the different possible cases. Here we recall some empirical laws that are valid for our case of interest, the upper surface of heated plate, as well as the symmetrical case of the lower surface of cooled plate. As flow conditions are generally not constant over a surface, only local heat transfer coefficient h and Nusselt number Nu can in general be defined; however, it is always possible to define average transfer coefficient $\bar{h}$ and corresponding Nusselt number by:

$$\bar{h} = \frac{1}{A_s} \int h \, dA_s \quad (14)$$

where A_s is the surface area of the zone where the average is to be defined.

Two regimes have been empirically identified [18] depending on the Rayleigh number[2] value, which give the average Nusselt number $\overline{Nu}$, and hence the average heat transfer coefficient $\bar{h}$, as a function of the Rayleigh number Ra which is defined as the product of the Grashof and Prandtl numbers:

$$\overline{Nu} = 0.54 \, Ra^{1/4} \;\; if \; 10^4 \lesssim Ra \lesssim 10^7 \quad (15)$$

$$\overline{Nu} = 0.15 \, Ra^{1/3} \;\; if \; 10^7 \lesssim Ra \lesssim 10^{11} \quad (16)$$

We will now determine which expression to use for the peculiar case of heat transfer by free convection at Ground Zero.

3.4 Numerical estimate of heat transferred by convection

As shown above, the average heat transfer coefficient $\bar{h}$ is determined through the average Nusselt number $\overline{Nu}$, which itself is expressed differently depending on the Rayleigh number Ra; it is therefore necessary to estimate first the value of this number. Let us recall the expression of the Rayleigh number:

$$Ra = \frac{g\beta \left(T_s - T_\infty\right) L^3}{\nu\alpha} \quad (17)$$

with the same notations as above. The numerical values are the following :

- gravity of Earth: $g \approx 9.81$ m.s^{-2} or $g \approx 10$ m.s^{-2} which is accurate enough for our purpose.
- volumetric thermal expansion of the air: considering it as an ideal gas and taking ambient temperature $T \approx 290$ K, we get $\beta \approx 3.45 \times 10^{-3}$ K^{-1}
- surface temperature T_s: obviously, it varies during cooling process. Shortly after the attacks, on September 16, 2001, temperatures as high as 1000 K have been measured using thermal imagery (Airborne Visible/Infrared Imaging Spectrometer). However, these temperatures were only "hot spot" temperatures and the visible surface (from above) of Ground Zero did not exhibit such high values on a large fraction of the area. As we are interested only in an order of magnitude value of the

[2] The Rayleigh number is the product of the Grashof and Prandtl numbers :

$$Ra = Gr\, Pr = \frac{g\beta \left(T_s - T_\infty\right) L^3}{\nu\alpha}$$

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temperature difference $T_s - T_\infty$, and as this difference slowly goes to zero during the cooling process, we will consider $T_s - T_\infty \sim 100$ K since 10 K would be obviously too small and 1000 K too large.

- characteristic length L: each of the Twin Towers (WTC1 and WTC2) had a roughly square horizontal section of $(64\,\mathrm{m})^2 \approx 4.10^3\,\mathrm{m}^2$ and WTC7 a slightly smaller but comparable footprint. According to thermal imagery hot zones were slightly larger than the respective footprints of the buildings. Therefore, we will take a characteristic length order of magnitude $L \sim 100$ m.
- kinematic viscosity of the air: at $T = 290$ K $\nu \approx 1.5 \times 10^{-5}$ m^2.s^{-1} and at $T = 390$ K has a larger value $\nu \approx 3 \times 10^{-5}$ m^2.s^{-1}. We can therefore take $\nu \sim 2 \times 10^{-5}$ m^2.s^{-1} as an order of magnitude value.
- thermal diffusivity of the air: at $T = 290$ K $\alpha \approx 2 \times 10^{-5}$ m^2.s^{-1}.

The above values give us:

$$\begin{aligned} Ra &= \frac{9.81 \times 3.45 \times 10^{-3} \times 100 \times 10^6}{2 \times 10^{-5} \times 2 \times 10^{-5}} \\ &\approx 8.5 \times 10^{15} \\ &\sim 10^{16} \end{aligned} \tag{18}$$

This very high Rayleigh number value is well above the range given for calculating the Nusselt number in equations 15 and 16. However some authors [19] have investigated thermal transport up to extremely high Rayleigh numbers ($Ra \sim 10^{17}$) and find no significant departure from the 1/3 exponent given in equation 16. Therefore, we will use this expression to estimate a numerical value of the average Nusselt number:

$$\overline{Nu} \approx 0.15\, Ra^{1/3} \approx 0.15 \times 10^{16/3} \approx 3.2 \times 10^4$$

And we finally compute an estimate of the average heat transfer coefficient $\overline{h}$ using the thermal conductivity of the air at 390 K, $k \sim 0.03$ W.m^{-1}.K^{-1}, and still $L \sim 100$ m:

$$\begin{aligned} \overline{h} &\approx \overline{Nu}\frac{k}{L} \\ &\approx 3.2 \times 10^4 \frac{0.03}{100} \\ &\sim 10\,\mathrm{W.m^{-2}.K^{-1}} \end{aligned} \tag{19}$$

Actually, $\overline{h}$ does not depend on L: since $\overline{Nu}$ scales as $Ra^{1/3}$ and Ra scales as L^3, $\overline{Nu}$ is proportional to L.

It could be argued that such a long theoretical development was not necessary to get a mere estimate of an amount of heat released, since engineers or architects are used to empirical numerical values for heat transfer coefficients. However, due to the unusual scale of the hot surface area, we considered safer to do so.

3.4.1 Estimating initial heat transfer rate

Let us now estimate the initial value P_0 of heat transfer rate, or thermal power, of the whole Ground Zero site short after the attacks, considering only free convection mechanism. As said before, heat transfer involves several mechanisms, namely convection (either free or forced), conduction and radiation. Each of these mechanisms gives a positive contribution to the global heat transfer rate, which means that we can only underestimate the amount of heat released if we consider, as will be done here, only one of them: free convection. This is precisely our aim: for the sake of our demonstration we do not need a correct estimate of the total amount of heat released at Ground Zero but only a lower estimate of it. It is important keeping this in mind for the following discussion.

In order to estimate the initial heat transfer rate, we need to estimate the initial temperature difference between the hot surface and the ambient air, as well as the corresponding heat exchange area. Of course, temperature was not uniform across the hot zones at Ground Zero, and a precise estimate of heat transfer should take into account such local variations; however, to get an order of magnitude of the initial thermal power we only need, since heat transfer laws by free convection are linear, to replace the real case by an equivalent one which is drastically simplified and reduces to a zone with a given uniform temperature and a given area.

Let us begin with the most imprecise guess: defining the initial equivalent uniform temperature. To do this, we will rely on actual temperature measurements of so-called "hot spots". Note that the temperature value in itself is not important *per se*, but rather it is the product of a temperature by a corresponding surface area which must be correctly determined, since it is this product which defines the heat transfer rate.

According to several publicly released documents [20, 21], three "hot zones" could be observed through thermal imagery (either airborne or satellite), corresponding respectively to WTC1, WTC2 and WTC7 debris. Surprisingly according to mainstream explanation of the collapses, and taking into account the very different levels of damage that suffered the three buildings (no plane hitting WTC7, and only minor fires compared to WTC1 and WTC2), all three collapse piles from WTC 1, WTC 2, and WTC 7 emitted infrared radiation with similar intensity as shown by a compilation of documents by A. Dreger [22], WTC1 and WTC2 hot zones having a slightly greater extent due to the larger spread of the debris during collapse.

It is also remarkable that no equivalent high temperatures / persisting fires phenomenon was encountered at the Pentagon site, although a similar attack with a similar passenger plane was supposed to occur. The Pentagon building was obviously very different from the WTC1, WTC2 and WTC7 buildings, but as the long lasting fires at Ground Zero were supposed to have been triggered by a large amount of jet fuel igniting office furniture, one should have expected not so large a difference between the WTC fires and the rapidly extinguished Pentagon fires.

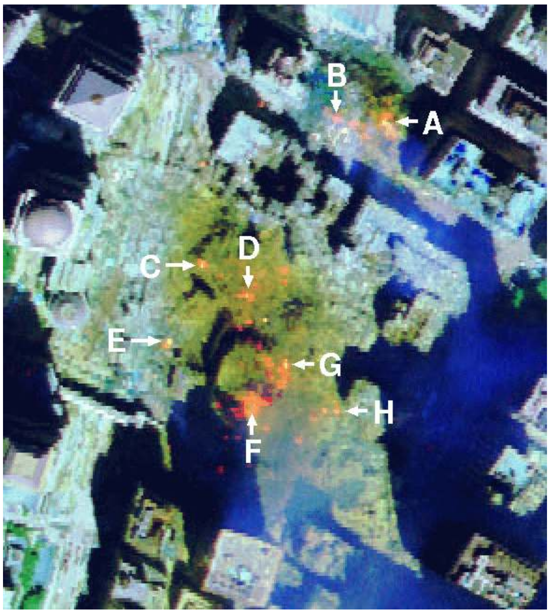

Figure 1: Some hot spots observed on September 16, 2001 with AVIRIS (Airborne Visible/InfraRed Imaging Spectrometer).
Source: United States Geological Survey[23].

According to USGS [23], determination of hot spots temperatures could be accurately performed using spectral analysis of the emitted radiation, as examplified in Fig. 2. On September 16, 2001, *i.e.* 5 days after the terrorist attacks, some surface temperatures as high as 1000 K could be measured, all the spots labeled by letters in Fig. 1 being at temperatures above 700 K.

Note that according to Planck's law, a black body at 1000 K already emits a noticeable part of its spectrum in the visible region (it is glowing red), although the maximum radiance, given by Wien's displacement law, occurs in the infrared zone at $\lambda_{max} \approx 2.9\,\mu\text{m}$.

A report from the Multidisciplinary Center for Earthquake Engineering Research (MCEER), "*Emergency response in the wake of the world trade center attack: The remote*

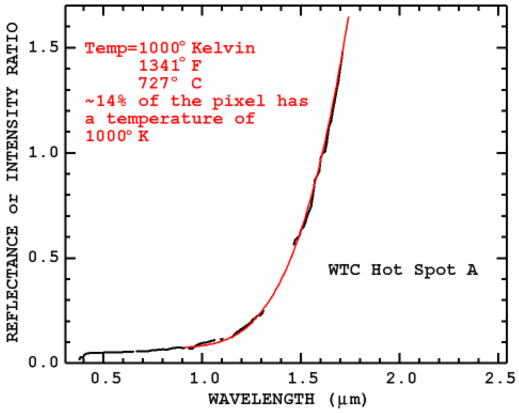

Figure 2: Example of IR radiation spectral analysis for surface temperature determination, as explained in [23]. The area at 1000 K (on September 16, 2001) covers 0.56 m^2 according to authors and is located in the WTC7 debris.

Figure 3: Infrared SPOT image, acquired three hours after the World Trade Center Attack on the 11th September 2001. From [21]. Note that although fires are already raging in WTC7 - which are supposed to be the cause of its future collapse according to NIST - no "hot spot" appears there, but only at WTC1 and WTC2 locations (in red).

sensing perspective" [21], gives several other thermal images which clearly show the same three hot zones, of roughly the same magnitude: WTC1, WTC2 and WTC7 debris. We can also note that this same paper displays (p. 17) the very first infrared image (1.58 - 1.75 μm wavelength) of Ground Zero, taken at 11:55 am on September 11, 2001 by a multispectral sensor of French satellite SPOT 4. This was roughly 3 hours after the WTC1 (08:46:40) and WTC2 (09:03:00) attacks and *before* the WTC7 collapse. Although this image has a poor spatial resolution of 20 m, it clearly shows only 2 "hot spots" at WTC1 and WTC2 locations, but no other one at WTC7 location. Therefore, although a note below the SPOT 4 image (see Fig. 3) in [21] reads "*Hotspots associated with fires raging at Ground Zero appear in red*", we suggest that the hot areas displayed in red are the *cause* of the fires rather that the fires themselves, which at that time extended well beyond the WTC1 and WTC2 footprints, and in particular in the WTC7 building still standing. Furthermore, the black (false) color of the plume rising from Ground Zero is consistent with water as a majority component of the plumes - which appears white in visible light - as water is opaque to infrared radiation. We will argue on this later in this paper.

On October 7th, 2001, a thermal image of Ground Zero (see Fig. 4) acquired by EarthData using a Raytheon airborne sensor ([21], Fig. 3.6, p. 22) still shows a very characteristic thermal pattern of roughly the same magnitude for all the three buildings (WTC1, WTC2 and WTC7) which suffered catastrophic failure on September 11, 2001. Unfortunately no temperature scale has been publicly released, but it should be emphasized again that although being architecturally very different and having suffered very different damages (no plane hitting WTC7), the resulting "thermal footprints" of the three collapsed buildings appear surprisingly similar.

According to U.S. National Institute of Standards and Technology (NIST), concerning

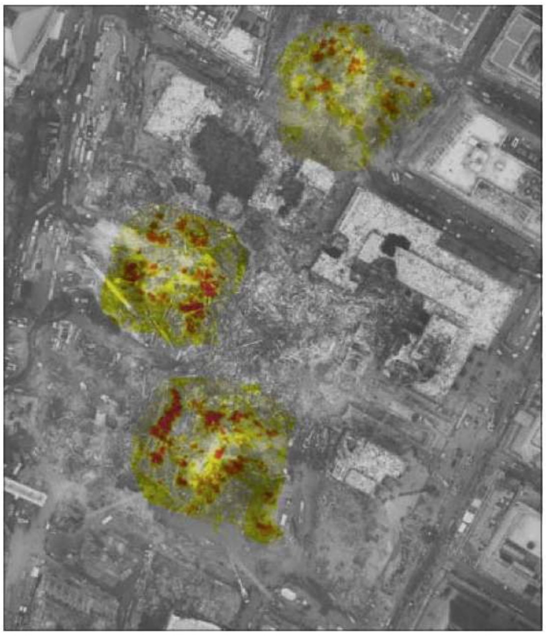

Figure 4: Thermal image of Ground Zero acquired by EarthData on the 7th October 2001, using a Raytheon airborne sensor. From [21], p. 22.

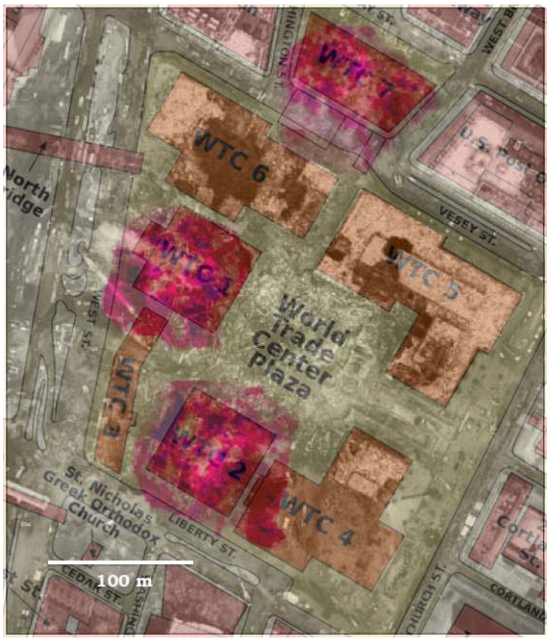

Figure 5: Superposition of thermal image obtained on October 7th, 2001 by EarthData shown in Fig. 4 and of a WTC map, with colours altered and scale added.

the Twin Towers (WTC1 & WTC2) "*The footprint of each tower was a square, about 210 ft. on a side*" (64 m)[24]; the footprint area for each tower was consequently about $S_{tower\,fp} = 64^2 \approx 4.1 \times 10^3$ m^2. Still according to NIST[25], WTC7 footprint was a trapezoid with a 140 ft. (42.7 m) width, and 2 parallel sides of 329 ft. (100.3 m) and 247 ft. (75.3 m); the footprint area of WTC7 was consequently about $S_{WTC7\,fp} = 42.7\frac{100.3+75.3}{2} \approx 3.75 \times 10^3$ m^2. The sum of the footprints areas for the 3 buildings that collapsed on September 11, 2001 was then about:

$$\begin{aligned} S_{total\,fp} &= 2S_{tower\,fp} + S_{WTC7\,fp} \\ &= 11.95 \times 10^3 \\ &\approx 12 \times 10^3 \,\mathrm{m}^2 \end{aligned} \tag{20}$$

As can be seen by superposition of thermal images and WTC site plans[3], "thermal footprints" were, in the first weeks, larger than the buildings footprints themselves. For instance, if one uses the EarthData thermal image given in Fig. 4, the image of Fig. 5, where colours have been altered for the sake of clarity and a scale added, can be obtained.

Any precise determination of the hot zones total area is difficult, but we want to recall here that it is anyway not our goal, since we only deal with orders of magnitude for our demonstration. Since the image shown in Fig. 5 was obtained nearly one month after the attacks, and since hot zones extend significantly further than the buildings footprints total area which is about 12×10^3 m^2, we consider that a good estimation of the hot zones total area for the initial heat release rate is $S_{hz} \approx 2 \times 10^4$ m^2.

As said before, this area in itself has little importance if it is associated with the wrong

[3] For instance the one that can be found on Wikipedia, itself based on NIST report drawings: https://en.wikipedia.org/wiki/World_Trade_Center_(1973-2001)

temperature difference ΔT; that is, taking a larger value with a lower ΔT can give the same heat release rate.

As cooling down of the 3 buildings debris took place over months, we do not need either to know a precise value of ambient temperatures; it began with rather mild ones in September ($T \approx 290\,K$ on September 11) but went down to much lower values during the following winter. We take the rather conservative[4] value of $T_a = 300\,K$ (27°C, or 80°F) for the following. As said before, several hot spots were measured at $1000\,K$ on September 16, 2001, 5 days after the attacks, which can be considered as almost an initial value; that makes a $\Delta T_0 = 700\,K$ difference with ambient temperature. Of course, there was by no means a uniform temperature on the whole "hot zone" defined previously, and we should *not* take this value as a good estimate for our simplified calculation, even in the order of magnitude perspective. On the other hand, since the average heat transfer coefficient $\overline{h} \approx 10\,\mathrm{W.m^{-2}.K^{-1}}$ we estimated before was based on theoretical results for horizontal plane surfaces, and since the hot wreckage zones were obviously not horizontal plane surfaces but exhibited a much larger contact surface with air, it is clear that we underestimate $\overline{h}$ in our model and should therefore not be too conservative in our mean temperature difference estimation. Somewhat arbitrarily, but keeping in mind that it should be enough for an order of magnitude estimation of initial heat release rate, we choose to take half of the maximum value given above for an equivalent hot zones temperature difference:

$$\Delta T_{0,hz} \approx 350\,\mathrm{K} \tag{21}$$

Hence we get the initial heat transfer rate, or thermal power associated with the three buildings hot zones:

$$\begin{aligned} P_0 &\approx S_{hz}\,\overline{h}\,\Delta T \\ &\approx 2 \times 10^4 \times 10 \times 350 \\ &\approx 7 \times 10^7\,\mathrm{W} \\ &\approx 70\,\mathrm{MW} \end{aligned} \tag{22}$$

[4]In the sense where it is rather high, and therefore will minimize the total amount of heat released in our calculations.

Again, this should not be considered as an accurate value but just as an indication that we should expect the actual initial thermal power to have been in the 100 MW range for the whole site, rather than in the 10 MW or 1 GW ranges.

3.4.2 Estimating characteristic cooling time

As said before, "*The underground fire burned for exactly 100 days and was finally declared "extinguished" on Dec. 19, 2001.*"[8]. Assuming an exponential decay for the temperature difference between hot surface and ambient air, which is a correct assumption if heat transfer laws are linear and ambient temperature a constant, we could take this 100 days value (a little more than 3 months) as a first estimation of the characteristic cooling time. However, as rather hot temperatures are needed to cause a fire, it might be considered as an underestimated value. Furthermore, among the publicly available thermal images of Ground Zero some of them[20], like the one shown in Fig. 6, still show a clearly noticeable hot zone on WTC1 location as far as February 12, 2002, that is 5 months after the terrorist attacks. Although no temperature scale is provided, the simple fact that some thermal signal significantly emerges from background noise on the image proves that Ground Zero is still cooling down and that the system is not at thermal equilibrium.**This is also a proof that fires were not the source for heat but the consequence of it**, since as stated above, the last underground fires were extinguished on December 19, 2001, almost 2 months earlier.

Consequently, and again somehow arbitrarily, but consistently with our order of magnitude approach, we chose an intermediate

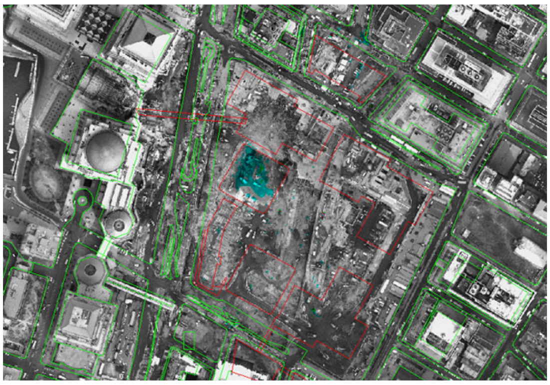

Figure 6: WTC – Thermal Imagery, February 12, 2002. New York State, Office for Technology (©2001) and EarthData International. From [20].

value of 4 months as characteristic time, or to express it in seconds:

$$\begin{aligned} \tau &\approx 4 \times 30 \times 24 \times 3600 \\ &\approx 10^7\,\mathrm{s} \end{aligned} \quad (23)$$

3.4.3 Estimating amount of heat released

Now, accordingly to equ. 4, we just have to multiply the initial heat transfer rate given in equ. 22 by the characteristic time given in equ. 23 to find an estimate of the total amount of heat released by free convection :

$$\begin{aligned} Q_{fc} &\approx P_0\,\tau \\ &\approx 7 \times 10^7 \times 10^7 \\ &\approx 7 \times 10^{14}\,\mathrm{J} \end{aligned} \quad (24)$$

Let us recall that the total amount of heat was released through several mechanisms: free convection in the air, forced convection (water), conduction and radiation. We chose to focus on free convection as it is probably the dominant mechanism in the problem, but each mechanism gives a positive contribution, so we can only underestimate the total value if we take into account only one mechanism. As above, this should be only considered as an indication that **the actual value ot total heat released at Ground Zero is in the 10^{15} J, or petajoule, range**:

$$Q_{GZ} \sim 10^{15}\,\mathrm{J} \quad (25)$$

This is, indeed, a really huge value which origin has to be questioned. As material supports for energy are well known, we will recall below some basic knowledge about the possible origins of energy, and especially the limits of any kind of chemical energy.

3.5 The physical limits of energy carriers

It is well known, even to undergraduate students, that there is a huge difference in mass for a given amount of energy released between conventional chemical energy sources like coal or oil and nuclear fuel used in nuclear reactors. Nuclear power plants need to be "refilled" with nuclear fuel only rarely (usually every 3 years, and only partly refilled), whereas most coal power plants must be designed with a railway track carrying millions of tons of coal annually. Let us recall below the physical origin of this difference.

3.5.1 Chemical energy carriers

Every undergraduate student in physical or chemical science knows that a chemical reaction is no more than a reorganization of electrons within bonds which tie atoms together; he or she knows also that energy levels of electrons involved in chemical bonds - considering only the strongest, covalent ones - do not exceed the electron-volt range. For instance, the H-H bond energy is 436 kJ/mol which translates into 7.24×10^{-19} J per bond or 4.52 eV. The N≡N triple bond, one of the strongest ones, has a 9.79 eV dissociation energy.

The energy involved in a chemical bond is always associated with some mass, but electrons are not by far the main mass carriers in atoms, since proton mass m_p and neutron mass m_n are much larger than electron mass m_e: $m_e \approx 9.1 \times 10^{-31}$ kg, $m_p \approx m_n \approx 1.67 \times 10^{-27}$ kg. We can therefore estimate the energy per unit mass of any chemical energy carrier if we can estimate how many protons and neutrons are associated with every electron involved in a covalent bond.

Since stable chemical compounds are always electrically neutral, and since electric charge in atoms is carried by electrons (for the negative charge) and by protons (for the positive charge), it can be inferred that for every electron involved in a covalent bond one can at least associate one proton in the corresponding atom. Since not all electrons are necessarily shared in chemical bonds, especially in heavy or moderately heavy atoms, some protons can also be present that will not be associated with any electron shared in a bond. Moreover, most atoms include neutrons in their nuclei, in a proportion which is generally of slightly more than one neutron for one proton (see Fig. 7).

Therefore, considering only an order of magnitude calculation, one can **estimate** the **upper** value $\left(\frac{E}{m}\right)_{max}$ of **any** chemical energy per unit mass by dividing one electron-volt by the mass of a nucleon:

$$\begin{aligned} \left(\frac{E}{m}\right)_{max}^{chemical} &\lesssim \frac{1\,\mathrm{eV}}{m_n} \\ &\approx \frac{1.6 \times 10^{-19}\,\mathrm{J}}{1.7 \times 10^{-27}\,\mathrm{kg}} \\ &\approx 10^8\,\mathrm{J.kg^{-1}} \\ &\approx 100\,\mathrm{MJ.kg^{-1}} \qquad (26) \end{aligned}$$

It should be emphasized that this value is slightly smaller than hydrogen Higher Heating Value[5] ($HHV_{H_2} \approx 142\,\mathrm{MJ.kg^{-1}}$) but

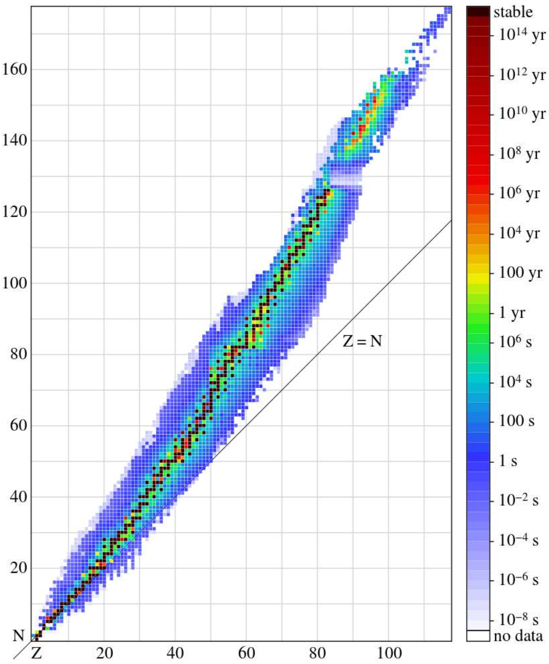

Figure 7: Plot of atomic isotopes (Z: number of protons, N: number of neutrons) colored by half life. From [26]

larger than all HHVs of any other fuel (for instance $HHV_{CH_4} \approx 55.5\,\mathrm{MJ.kg^{-1}}$), hydrogen being the one and only molecule where each atom consists only in one proton and one electron, which is the most favourable case.

Actually this upper limit calculation assumes that there are no neutrons associated with protons (otherwise we would have to divide the result by 2 at least) and that every electron is involved in a covalent bond, two conditions that are fulfilled only by hydrogen gas. If we exclude this very special case, we should rather take the following limit:

$$\left(\frac{E}{m}\right)_{max}^{chemical\ excl.\ H_2} \lesssim 5 \times 10^7\,\mathrm{J.kg^{-1}} \quad (27)$$

It should be emphasized also that such a mass constraint is entirely independent of the chemical nature of the compound (com-

[5] For practical reasons engineers define Higher Heating Values (HHVs) and Lower Heating Values (LHVs) for combustibles. The former equals the thermodynamic heat of combustion (or enthalpy change) whereas the latter does not take into account energy released by water condensation. For our purpose this difference does not actually matter.

bustible[6], explosive[7], or even food[8]) and of technological refinements, that can act on energy release rate (*i.e.* power) but *not* on energy per unit mass.

A lot of scientific and technological excitement has occured in the last decades in the field of so-called "nano-technologies"; however, since the very principle of such technologies is to finely divide matter down to the molecular level, only the surface/volume ratio of the chemical compounds can be increased and consequently the speed of a chemical - or physical - process, which depends heavily on this ratio. For instance, the battery industry has made a significant leap in energy content per unit mass when replacing lead-acid batteries by Li-ion batteries, thanks to the small mass of lithium compared to lead; but it has made much more dramatic improvements in power per unit mass using finely divided electrodes, and can produce now on an industrial scale power batteries delivering as much as several kilowatts per kg[9], which are very useful for electric or hybrid cars, or even motorcycles used in drag racing competitions.

Furthermore, we must keep in mind that in most cases, when expressing chemical energy per unit mass this mass does *not* include all the reactants, since energy comes from an exothermic redox reaction and that the oxidizing agent is oxygen contained in the atmosphere, which is considered as free and unlimited. For instance, the combustion of

Figure 8: Food industry provides energy content on the packaging for its products. Here, "Servietten Knödel" from Germany, with a 0.78×10^7 J.kg^{-1} energy content.

[6] The well-known *"ton oil equivalent"* or *toe* equals 42 GJ, hence the energy per unit mass of oil is 4.2×10^7 J.kg^{-1}.

[7] It is of common use to express energy released by an explosion in "TNT equivalent"; a ton of TNT equals by convention 4.184 GJ, hence the energy per unit mass of TNT is 4.184×10^6J.kg^{-1}.

[8] Food industry indicates on every packaging the amount of energy for a given mass (in Europe, usually for 100 g); it is therefore very easy to check that food energy content is also in the 10^7J.kg^{-1} range.

[9] For instance A123 Systems announces "over 4000 W/kg" for its AHP14 Lithium-ion prismatic cell: http://www.a123systems.com/prismatic-cell-ahp14.htm

hydrogen can be written as:

$$H_2 + \frac{1}{2}O_2 \rightarrow H_2O$$

which means that for every molecule of hydrogen, weighing 2 grams per mole, one atom of oxygen, weighing 16 grams per mole, is needed. Therefore the mass of all reactants is $2+16 = 18$ grams per mole, whereas the mass of hydrogen molecule is only 2 grams per mole, *i.e.* 9 times less. Then if we take the ratio of energy to mass using the total mass instead of the mass of hydrogen alone, we must divide the previously given value by 9 and we get the modified higher heating value $(\mathrm{HHV}_{\mathrm{H_2}})^* \approx 16\,\mathrm{MJ.kg^{-1}}$.

The same calculation for methane (80 grams instead of 16) will give $(\mathrm{HHV}_{\mathrm{CH_4}})^* \approx 11\,\mathrm{MJ.kg^{-1}}$ whereas $(\mathrm{HHV}_{\mathrm{CH_4}}) \approx 55.5\,\mathrm{MJ.kg^{-1}}$, and for pure carbon (44 grams instead of 12), $(\mathrm{HHV}_{\mathrm{C}})^* \approx 9\,\mathrm{MJ.kg^{-1}}$ whereas $(\mathrm{HHV}_{\mathrm{C}}) \approx 33\,\mathrm{MJ.kg^{-1}}$.

This is the reason why, when a chemical compound can release energy by itself without needing an additional reactant such as oxygen, its energy content per unit mass is much lower than that of usual fuels. This is the case for explosives: one of the most famous ones, trinitrotoluene (TNT), has an energy content of roughly $4.2 \times 10^6\,\mathrm{J.kg^{-1}}$, that is one tenth that of oil.

In conclusion, although we computed in Equ. 26 a **maximum** order of magnitude of chemical energy per unit mass, which is in the $10^8\,\mathrm{J.kg^{-1}}$ range for hydrogen, this is a very special case of little relevance if we deal with heat coming from underground fires, since it is rather obvious that such a heat can not originate from the combustion of pure, ideal fuels like H_2 (or CH_4, or even oil...) but only from complex, solid materials that burn only partially. We will consequently retain a more practical, effective order of magnitude for chemical energy per unit mass that could be released at Ground Zero by any material:

$$\left(\frac{E}{m}\right)_{max}^{GZ} \approx 10^7\,\mathrm{J.kg^{-1}} \qquad (28)$$

3.5.2 Nuclear energy carriers

Just as chemical energy comes from modification of bonds between atoms, nuclear energy comes from modification of bonds between nuclei; but as any undergraduate student knows, the energy levels of these bonds are much higher, on the order of a million times greater, and this is the reason why atoms are stable. Binding energy of a nucleus can also be expressed, according to Einstein's equation $E = mc^2$, as a mass difference or "mass defect" between the mass of a nucleus and the sum of the masses of the nucleons of which it is composed [27].

It is useful to express the binding energy per nucleon as a function of the number of nucleons in the nucleus; the obtained curve exhibits a plateau in the vicinity of iron (^{56}Fe) as shown on Fig. 9. ^{62}Ni has actually the largest binding energy per nucleon (see for instance [28]) but ^{56}Fe has the least average mass per nucleon, having a smaller neutron/protons ratio than ^{62}Ni. It follows that some energy can be released when heavy nuclei split into parts (nuclear fission), or when light nuclei combine together (nuclear fusion). Both solutions have been extensively studied since World War II, for military and civilian purposes, and lead to an energy gain on the order of 1 MeV per nucleon, as can be seen on the curve on Fig. 9.

Let us give two examples than can be found in many textbooks:

- fission reaction, starting from ^{235}U:

$$\begin{aligned} {}^{235}\mathrm{U} + {}^{1}\mathrm{n} &\rightarrow {}^{90}\mathrm{Kr} + {}^{143}\mathrm{Ba} + 3\,{}^{1}\mathrm{n} \\ &\rightarrow {}^{90}\mathrm{Zr} + {}^{143}\mathrm{Nd} \\ &\quad +3\,{}^{1}\mathrm{n} + 8\,\mathrm{e}^{-} \end{aligned} \qquad (29)$$

  The energy released is $\Delta E = 198$ MeV, for 235 nucleons (236 if we take into

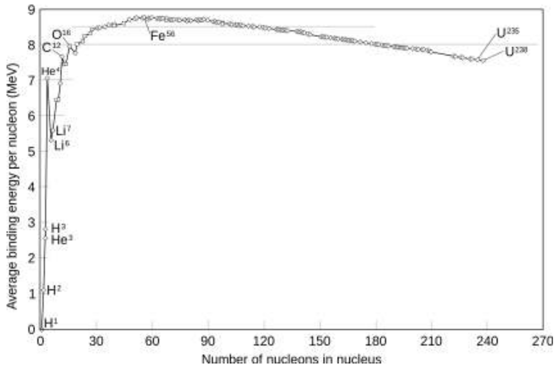

Figure 9: Average binding energy per nucleon in MeV against number of nucleons in nucleus, for relatively abundant isotopes. From [29]

account the incoming neutron), which gives roughly 0.84 MeV per nucleon.

- fusion reaction with deuterium:

$$2\,{}^2\mathrm{H} \rightarrow {}^3\mathrm{H} + {}^1\mathrm{H} \tag{30}$$

The energy released is $\Delta E = 4.54$ MeV, for 4 nucleons, which gives 1, 13 MeV per nucleon.

So in both cases, fission or fission, energy released per unit mass is on the order of magnitude of 1 MeV divided by the mass of a nucleon, which is 10^6 times greater than what can be achieved with chemical energy:

$$\left(\frac{E}{m}\right)^{nuclear}_{max} \lesssim \frac{1.6\times10^{-13}\,\mathrm{J}}{1.7\times10^{-27}\,\mathrm{kg}} \approx 10^{14}\,\mathrm{J.kg^{-1}} \approx 100\,\mathrm{TJ.kg^{-1}} \tag{31}$$

However, in contrast to the case of chemical energy where complete combustion of a fuel will give energy per unit mass on the order of what has been determined in Equ. 26 or a little less, this theoretical limit is of little relevance for practical applications of nuclear energy, be they nuclear reactors or nuclear bombs. First because not all the "nuclear fuel" will react in the process, and second because a lot of material which does not take part at the reaction is necessary to build a working nuclear device. Hence the practical energy/mass ratio will be much lower, as we will see on some examples below, but the difference with chemical energy/mass ratio will still remain huge.

For instance, the first practical thermonuclear device to be detonated by the USA, during the Castle Bravo test on March 1, 1954 on the Bikini Atoll, had a mass of roughly 10.7×10^3 kg and a "yield", or quantity of energy released, of 15 megatons of TNT[10], which is approximately 63×10^{21} J. That gives an energy/mass ratio of:

$$\left(\frac{E}{m}\right)_{CB} \approx \frac{63\times10^{15}}{10.7\times10^3} \approx 5.9\times10^{12}\,\mathrm{J.kg^{-1}} \tag{32}$$

This is rather far from the value given in Equ. 31, but several orders of magnitude higher than the $1.42\times10^8\,\mathrm{J.kg^{-1}}$ value for the "best" chemical energy source known, hydrogen.

And even much older devices, like the historical fission bombs that exploded over Hiroshima and Nagasaki on August 6 and 9, respectively, in 1945, outperformed conventional, chemical bombs by several orders of magnitude also. Let us recall their characteristics:

- for "Little Boy", the ^{235}U bomb that was dropped on Hiroshima, the mass was 4400 kg and the yield 15 kT of TNT, or 63×10^{12} J, which gives:

$$\left(\frac{E}{m}\right)_{LB} \approx \frac{63\times10^{12}}{4.4\times10^3} \approx 1.4\times10^{10}\,\mathrm{J.kg^{-1}} \tag{33}$$

- for "Fat Man", the ^{239}Pu bomb that was dropped on Nagasaki, the mass was 4700 kg and the yield 21 kT of TNT, or 88×10^{12} J, which gives:

$$\left(\frac{E}{m}\right)_{FM} \approx \frac{88\times10^{12}}{4.7\times10^3} \approx 1.9\times10^{10}\,\mathrm{J.kg^{-1}} \tag{34}$$

[10] For an expected yield of "only" 5 megatons.

Note that the bomb contained a 6.2 kg plutonium mass, of which approximately 1 kg underwent nuclear fission. If we consider only this one-kilogram mass, we end with $8.8 \times 10^{13} \approx 10^{14}$ J.kg^{-1}, which is the value given in Equ. 31.

Even though these values were more than 100 times lower than the one achieved during Castle Bravo nuclear test, they never could have been reached with any conventional chemical explosives, TNT being, at 4.2×10^{6} J.kg^{-1}, more than 3000 times heavier for the same energy released.

As an intermediate value, we can also compute the $\frac{E}{m}$ ratio for a 100-kt nuclear explosive, weighing 5 metric tons, given as an example in Teller *et al.* book *"The Constructive Uses of Nuclear Explosives"* [30], p. 129:

$$\begin{aligned}\left(\frac{E}{m}\right)_{Teller\,et\,al.} &\approx \frac{100 \times 4.18 \times 10^{12}}{5000} \\ &\approx 8.4 \times 10^{10}\,\text{J.kg}^{-1} \\ &\sim 10^{11}\,\text{J.kg}^{-1} \qquad (35)\end{aligned}$$

This is the reason why nuclear weapons are such a strategic asset, since a single plane can carry a bomb powerful enough to cause massive devastation, and since miniaturized versions, so-called "tactical nuclear weapons", can cause enormous damage compared to their chemical counterparts if used on battlefields, or even for terrorist actions as they are easy to conceal inside a small vehicle or even carried by men. The smallest nuclear weapons ever reported had indeed a smaller yield than that of the largest conventional (chemical) ones: for instance, the USA manufactured the Davy Crockett Weapon System that had a yield between 10 and 20 tons of TNT (42 to 84 GJ), whereas the Aviation Thermobaric Bomb of Increased Power (ATBIP) developped recently (2007) by Russia has a claimed yield of 44 tons of TNT. But of course, the masses of the devices are not comparable, with 23 kg for the Davy Crockett and 7 100 kg for the ATBIP. Note that the Davy Crockett is, at $\frac{E}{m} \approx 3 \times 10^{9}$ J.kg^{-1}only, comparatively "heavy" for a nuclear bomb.

3.5.3 Conclusion regarding Ground Zero heat source

Taking back the result obtained in Equ. 25 for heat released at Ground Zero, we recall that this energy was in the petajoule range, or 10^{15} J. Combining this result with Equ. 28 and 35 (for a more realistic case than 31), we end up with the following mass requirements depending on the nature of energy source:

- for a chemical energy source,

$$\begin{aligned} m_{chemical} &\sim \frac{10^{15}}{10^{7}} \\ &\sim 10^{8}\,\text{kg} \qquad (36)\end{aligned}$$

  which can be also expressed as 100 000 metric tons.

- for a nuclear energy source comparable with a nuclear explosive such as the one cited by Teller *et al.*,

$$\begin{aligned} m_{nuclear} &\sim \frac{10^{15}}{10^{11}} \\ &\sim 10^{4}\,\text{kg} \qquad (37)\end{aligned}$$

  or 10 metric tons.

The masses of WTC1, WTC2 and WTC7, most of which was structural steel and light concrete used in the floors, and therefore *not* combustible, were in the 10^{8} kg range: for instance NIST claims that each of the Twin Towers used 100 000 metric tons of structural steel (see [31], p. 55). Some authors (see [32]) conclude after a detailed analysis of the materials involved in the building that the in-service mass of WTC1 was about 2.9×10^{8} kg (290 000 metric tons), a figure consistent with the mass per floor unit area of similar and contemporaneous buildings like John Hancock Center (1969) or Sears Tower (1973).

With this simple calculation, we can therefore rule out any chemical origin of the heat released at Ground Zero during the months following September 11, 2001 terrorist attacks: that would have required a significant fraction of the buildings masses to be combustible, which is absurd. As we already mentioned in subsection 3.1, heat was not a *consequence* of fires but the *cause* of them, because the second law of thermodynamics precludes heat to flow spontaneously from the lower temperatures to the higher ones, which consequently prevents buildings fires to melt (or worse vaporize) steel. Here, we have demonstrated that the first law of thermodynamics also leads to the same conclusion.

It is therefore impossible that the cause for underground fires was some pyrotechnic compound like thermite or "nano-thermite", as some authors have suggested (see for instance [33]). We do *not* claim that such compounds were not used at all in the whole process; we only claim that they cannot explain the amount of heat that was released after the attacks.

Furthermore, it is *also* impossible that WTC destruction was done using miniaturized nuclear bombs planted inside the buildings, as some have suggested, first because of the lack of characteristic effects of aerial nuclear explosions, and second because the energy released during the cooling process of Ground Zero corresponds to energies released by big nuclear weapons, not small ones - or a large number of them would have been needed. As there were three distinct events (WTC2, WTC1 and WTC7 collapses, in chronological order), we claim that 3 nuclear bombs of respectable size were detonated deep underground. We discuss in appendix B how this surprising conclusion can be more easily understood in the technical context of the 1960s', since it is quite obvious that burying three big nuclear devices deep in the ground under three skyscrapers could not be a "classical" terrorist operation but only the opportunistic use of a built-in feature.

As it is usual to express the "size" of a nuclear bomb in TNT equivalent, let us translate our figure into this non-standard unity. A kiloton of TNT equals, by convention, 4.184×10^{12} J. We estimated (Equ. 25) that the total amount of heat released at Ground Zero was on the order of $Q_{GZ} \sim 10^{15}$ J. This translates into:

$$Q_{GZ} \sim \frac{10^{15}}{4.18 \times 10^{12}} \sim 240 \,\text{kilotons of TNT} \tag{38}$$

Assuming - because of the similar "thermal footprints" of the three collapses as can be seen in Fig. 4 - an equal "size" for all three bombs, this translates to 80 kilotons per bomb; however, although it is by far the major part, not all the energy of a deep underground nuclear explosion converts into heat.

As we deal only with order of magnitude calculations, we propose then, using this conventional unity and keeping only a power of ten expression, that **3 deep underground nuclear explosions occurred on September 11, 2001 under the World Trade Center site, each of them at least of 50 kt and more probably on the order of 100 kt of TNT**. This is comparable to the Sedan nuclear test already mentioned above, which yield was 104 kt of TNT. However, as the device was disposed in the desert alluvium in the case of Sedan (at a depth of 194 m), it is clear that the effect of the explosion was very different in New York, since every skyscraper needs to be anchored in a lithified rock for obvious stability reasons.

Note that this simple energetic argument does *not* prove in itself that the origin of the heat at Ground Zero was the explosion of underground nuclear *bombs*; it just proves that the only known type of energy able to do that was nuclear, not chemical. But as it is quite obvious that only an explosion (and not a progressive release of energy that occurs with a nuclear reactor) could have such a dramatic effect, and as it is also quite obvious that there was no aerial[11] nuclear explosion in New York on September 11, 2001, we need now to check if underground nuclear blasts can explain satisfactorily what was observed there. Let us begin with a brief introduction to the effects of underground nuclear explosions.

4 Nuclear explosions as an engineering tool

4.1 Basic knowledge about underground nuclear explosions

Underground nuclear explosions have been extensively studied since November 29, 1951 when the USA performed the first underground nuclear weapon test within the framework of the *Jangle* program at the Nevada Test Site (see for instance [34], p. 8). Such experiments were initially done for military purposes but were also eventually conducted for civilian ones (see for instance [35]), mainly civil engineering and energy production. In the USSR underground nuclear testing began a decade later in the Kazakh Socialist Soviet Republic known today as Kazakhstan, and in November 7, 1961, France performed its first underground nuclear test at the Reggane site in the Sahara desert, Algeria.

Numerous studies have been made since about underground nuclear explosions, and although the most detailed ones are probably kept secret for obvious military reasons, enough documents are publicly available to give us a pretty good idea of the overall picture, which is all what we need to address our investigation. Most of the papers are of the technical report type, emphasizing experimental results like cavity size, seismic signature etc., but do not give a good understanding of the underlying physics, which departs significantly from the one encountered with "ordinary" chemical explosions. However, it appears that at least[12] one book stands out that can replace this very specific experimental field in the general framewok of physics: *The Constructive Uses of Nuclear Explosives*, by Edward Teller, Wilson K. Talley, Gary H. Higgins and Gerald W. Johnson [30]. Some textbooks on nuclear explosions are also nowadays freely available, like the classical "*The Effects of Nuclear Weapons*" edited and compiled by Samuel Glasstone and Philip J. Dolan [36]. It is worth noting that the book written by Teller *et al.*, published in 1968, was intended to promote - as its title says - non-military use of nuclear explosions, mainly for large and energy-consuming civil engineering.

It is nowadays almost forgotten that such uses can exist, and were actually extensively investigated at the beginning of the "nuclear era", especially in the 1960's, both by the USA and by former USSR[13]; most people can only cite today, as civilian uses of nuclear energy, electricity production - be it in big stationary power stations or in smaller mobile ones used in nuclear-powered ships [37] or submarines. We address briefly this technoscientific collective amnesia in Appendix B, where we recall some of the most striking peaceful engineering projects based on nuclear explosives.

In the following subsections, we will first give some overall picture of the kind of physics involved in nuclear underground explosions, mainly relying on [30], and then deduce from it, and from comparisons with experimental data, some numerical estimates of the phe-

[11] In aerial explosion we include also explosions inside a building, even at underground levels: among of the numerous effects of such explosions are the "fireball" and, of course, a tremendous acoustic shock wave.

[12] We have not made an extensive bibliographical research on this subject.

[13] During 1960s and 1970s, both USA and USSR conducted peaceful nuclear explosions programs, named *Operation Plowshare* in the USA and *Nuclear Explosions for the National Economy* (Мирные ядерные взрывы в СССР) in the USSR.

nomena produced by nuclear devices consistent with our energy release estimate.

4.1.1 Some specificities of nuclear explosions physics

As seen earlier (see subsection 3.5.2), nuclear energy is, per unit mass, roughly 10^6 larger[14] than chemical energy, which also implies that it is contained in a much smaller volume for the same energy. Moreover, in a nuclear explosion, energy is released in a typical timescale $\tau_{nuc} \sim 10^{-6}$ s, whereas for chemical explosives it is a much longer timescale $\tau_{ch} \sim 10^{-3}$ s. These two simple facts account for the extremely high temperatures (megakelvin range) and pressures (billions of atmospheres, or $\sim 10^{14}$ Pa) encountered in the first stages of nuclear explosions, around the place where the device was triggered. The temperatures obtained are high enough to strip most electrons from their orbitals and turn everything into a plasma, which for aerial nuclear explosions results in the well-known "fireball".

Although such values are far above most of experimental physics knowledge, they lead to some theoretical simplifications that are worth pointing out, for two reasons: they are a good illustration of the simplicity and power of physics, and they provide us a safe way to make numerical estimates in a pressure and temperature range where no probe can resist. As Teller *et al.* point out in [30], nuclear explosions physics bears some similarities with astrophysics, since in the vicinity of the shot point the states of matter are the same, and chemical differences are no longer relevant since everything, including rocks, is turned into a plasma.

A first, somewhat counter-intuitive result is that vaporized rock in the vicinity of the explosion can be treated as an ideal gas. Students are generally told to be cautious with ideal gas laws, especially when dealing with very high pressures. How then is it possible that tremendous pressures generated by a nuclear explosion do not make this piece of advice valid? To understand that, one has to remember that an ideal gas is merely a collection of independent particles which energy is purely of thermal nature, *i.e.*, a collection of particles which interaction energy can be neglected compared to their kinetic energy.[15] This is precisely the case for the collection of electrons and ions forming a plasma just after[16] a nuclear explosion, which temperature and pressure can be derived quite easily following the arguments given by Teller *et al.*, that we shall summarize below.

Derivation of initial temperature

The energy released ("yield" of the device, in common language) is known, as well as the volume of the bomb, within which one can assume this energy is initially distributed. Teller *et al.* give as an example, for a 100-kt nuclear explosive, a cylindrical canister 1 m in diameter and 3 m long, which volume is:

$$V = \pi R^2 h \approx 2.36\,\mathrm{m}^3 \tag{39}$$

Since the energy released is $E = 100 \times 4.18 \times 10^{12} \approx 4.18 \times 10^{14}$ J, this accounts for an initial volumic energy density:

$$\begin{aligned} \mathscr{E} &\approx \frac{4.18 \times 10^{14}}{2,36} \\ &\approx 1.77 \times 10^{14}\,\mathrm{J.m^{-3}} \end{aligned} \tag{40}$$

Note that an energy divided by a volume is homogeneous to a pressure, and this simple result can already give us the right order of

[14] Or, as seen above, for a technically feasible device $\sim 10^4$ times larger, which is much less but still enough to make a huge difference.

[15] Actually, the very concept of an "ideal gas" is much more general than an idealization or real gases, since some applications of it can be found in solids, for instance in polymer science [38] where it is a key to understanding elasticity of rubber or gels.

[16] In the few microseconds following the chain reaction triggering.

magnitude for the initial pressure level, in pascals (translating into $\sim 2 \times 10^9$ atm).

Now this total energy density can be divided in 2 contributions, namely a material and a radiative one:

$$\mathscr{E} = \mathscr{E}_{mat} + \mathscr{E}_{rad} \tag{41}$$

The material part is the translational kinetic energy density of particles at temperature T in a 3-dimension space, that is, in accordance with equipartition of energy:

$$\mathscr{E}_{mat} = \frac{3}{2}\, n\, k_B T \tag{42}$$

where n is the number of particles per unit volume and k_B Boltzmann constant.

The radiative part is the energy density of a photon gas which can be expressed as:

$$\mathscr{E}_{rad} = \frac{8\pi^5}{15}\,\frac{k_B^4}{(hc)^3}\, T^4 \tag{43}$$

where $h \approx 6.626 \times 10^{-34}$ J.s is Planck constant and $c \approx 2.998 \times 10^8$ m.s^{-1} the speed of light in vacuum; or more simply, using Stefan-Boltzmann constant $\sigma = \frac{2\pi^5}{15}\frac{k_B^4}{h^3c^2} \approx 5.67 \times 10^{-8}$ W.m^{-2}.K^{-4}:

$$\mathscr{E}_{rad} = \frac{4\sigma}{c}\, T^4 \tag{44}$$

Whereas both σ and c in Equ. 44 are well known constants, one has to precise the value of n in Equ. 42 which depends on the particular case studied. As stated before, we expect temperatures high enough to strip electrons from their orbitals and produce a plasma consisting only in electrons and nuclei[17]. Hence, the particles we must count in this ideal gas are the electrons and the nuclei[18]. Now, to determine how many particles per unit volume are present in the initial volume of the canister we can use its volumic mass, and consider the fact that mass comes almost exclusively from nucleons. According to the figures given by Teller *et al.* we have a mean volumic mass:

$$\begin{aligned} \rho &\approx \frac{5000}{2.36} \\ &\approx 2.12 \times 10^3\,\text{kg.m}^{-3} \end{aligned} \tag{45}$$

The mass of a nucleon - be it a neutron or a proton - is $m_n \approx 1.67 \times 10^{-27}$ kg, therefore the number of *nucleons* per unit volume is:

$$\begin{aligned} n' &\approx \frac{\rho}{m_n} \\ &\approx \frac{2.12 \times 10^3}{1.67 \times 10^{-27}} \\ &\approx 1.27 \times 10^{30}\,\text{m}^{-3} \end{aligned} \tag{46}$$

Now we do not look for the number of nucleons but for the total number of particles, which are nuclei and electrons. As stated previously in subsection 3.5.1, there are approximately as many neutrons as protons in a nucleus (slightly more neutrons) except obviously for hydrogen, and there are exactly as many protons as electrons for the sake of electrical neutrality. From this simple information we can infer that the number of electrons per unit volume is roughly half that of nucleons:

$$n_e \approx \frac{n'}{2} \tag{47}$$

But for an order of magnitude calculation, given that most chemical elements in the nuclear explosive canister will not be of very low atomic number, we can even consider that the number of *nuclei* - not nucleons! - is negligible compared to the number of electrons[19], so that the desired particle density

[17] We use here the word "nuclei" instead of "ions" to emphasize that most electrons are free; however, ionization can still be incomplete.

[18] It is worth pointing out here that in the ideal gas model, every particle has the same mean kinetic energy regardless of its mass. Therefore, electrons and nuclei, although having extremely different masses, give the same contribution to kinetic energy and must be considered equally.

[19] For instance, the chemical element sulfur, in its alpha form, has a volumic mass at room temperature and ambient pressure of 2.07×10^3 kg.m^{-3}, roughly equal to the volumic mass we just calculated for the nuclear explosive canister. Sulfur has an atomic number of 16, therefore counts 16 protons and 16 electrons. Neglecting to count the nuclei in a plasma obtained when all electrons are separated from their nuclei will then give a $1/16 \approx 0.06$ relative error, which is more than acceptable for our purpose.

is close to the electrons density:

$$\begin{aligned} n &\approx n_e \\ &\approx \frac{\rho}{2m_n} \\ &\approx 6.34 \times 10^{29}\,\mathrm{m}^{-3} \end{aligned} \tag{48}$$

In short, we only count the electrons in the ideal gas particles but we only consider nucleons mass. As pointed out by Teller *et al.* in their book (p. 131), this is equivalent to having a molecular ideal gas with an effective molar mass $M_{eff} \approx 2\,\mathrm{g.mol}^{-1}$, like deuterium.

We now have all numerical data to solve Equ. 41 for temperature; let us write it explicitly:

$$\mathscr{E} = \frac{3}{2}\, n\, k_B T + \frac{4\sigma}{c}\, T^4 \tag{49}$$

or numerically, in SI units:

$$\mathscr{E} \approx 1.31 \times 10^7\, T + 7.57 \times 10^{-16}\, T^4 \tag{50}$$

Although extremely tedious to solve analytically[20], this equation can easily be solved numerically for temperature; the above values will give, for $\mathscr{E} \approx 1.77 \times 10^{14}\,\mathrm{J.m}^{-3}$ as we found in Equ. 40, a temperature:[21]

$$T_{init} \approx 1.22 \times 10^7\,\mathrm{K} \tag{51}$$

Derivation of initial pressure

We have already got thanks to Equ. 40 a crude estimate of the initial pressure, which is in the 100 TPa range (10^9 atm). However, since photons and matter behave differently for generating pressure it is better to give the correct expression as Teller *et al.* do:

$$p = \frac{2}{3}\mathscr{E}_{mat} + \frac{1}{3}\mathscr{E}_{rad} \tag{52}$$

The reason is that pressure exerted on a cavity walls comes from the momentum change upon collision of the particles it contains, and kinetic energy for particles with mass m is expressed as $E_{ke} = \frac{1}{2}mv^2 = \frac{1}{2}pv$ whereas it reads for massless photons of velocity c: $E_{ke} = pc$. Consequently, energy density will be $\mathscr{E} = \frac{3}{2}p$ for material particles and $\mathscr{E} = 3p$ for photons. In other words, for a given energy density, photons exert one half the pressure a material gas would exert.

Let us give Equ. 52 in numerical form:

$$p \approx 8.75 \times 10^6\, T + 2.52 \times 10^{-16}\, T^4 \tag{53}$$

Reporting T_{init} value found above gives an initial pressure:[22]

$$p_{init} \approx 1.13 \times 10^{14}\,\mathrm{Pa} \tag{54}$$

which comes mainly from material pressure:

$$p_{mat} \approx 1.07 \times 10^{14}\,\mathrm{Pa}, \quad p_{rad} \approx 6 \times 10^{12}\,\mathrm{Pa}$$

This tremendous pressure ($\sim 10^9$ atm) generates a shock wave which travels at supersonic speed and destroys every material integrity around the explosion point, long before heat can diffuse; we will discuss this aspect more precisely below.

Shock wave effects on materials

We will not in the following present a detailed description of shock waves, which can be found in many textbooks, but only recall some basic facts about shockwaves in general, and what kind of effects they can have in the specific case of an underground nuclear explosion generating initial pressures as high as the ones derived above.

[20] Some online equation solvers (*e. g.* numberempire.com) will give the four solutions in an instant, only one of which is physically acceptable. But its mathematical expression is itself particularly cumbersome and impractical.

[21] Note that with the same numerical values and approximations, Teller *et al.* give a slightly different temperature $T_{init} \approx 1.37 \times 10^7\,\mathrm{K}$, and a slightly different numerical equation $\mathscr{E} \approx 1.32 \times 10^7\, T + 7.65 \times 10^{-16}\, T^4$. However, solving numerically this equation for the same total energy density $\mathscr{E} \approx 1.77 \times 10^{14}\,\mathrm{J.m}^{-3}$ (100 kt in a cylindrical canister 3 m long and 1 m in diameter) still gives $T_{init} \approx 1.22 \times 10^7\,\mathrm{K}$. Anyway, it does not change the important result, that is an initial temperature in the 10 MK range.

[22] Teller *et al.* find, here, $p_{init} \approx 1.15 \times 10^{14}\,\mathrm{Pa}$

A longitudinal wave travels in a medium of volumic mass ρ with a speed:

$$v = \frac{1}{\sqrt{\rho\chi_S}} = \sqrt{\left(\frac{\partial p}{\partial \rho}\right)_S} \tag{55}$$

where the subscript S indicates an isentropic transformation. Shock waves are intrinsically irreversible and thus *not* entropy-conservative, however the above equation can explain why any compression wave of high enough amplitude should evolve towards a shock wave, *i.e.* an almost discontinuous pressure variation that travels through the material.

In the crest of a wave[23], pressure is higher and so is temperature, since the transformation is adiabatic. As a result, the crest goes faster than the rest of the wave, which ends in a sawtooth-shaped signal instead of the initial sinusoïdal one. This can easily be understood in the particular case of ideal gases where the speed of sound can be expressed as $v = \sqrt{\frac{\gamma RT}{M}}$, where temperature T appears explicitly, but although different the physics remains qualitatively the same for most materials.

On the other hand, a sawtooth-shaped signal can be decomposed in Fourier analysis as a sum of frequencies which are multiples of the initial characteristic frequency of the sinusoidal signal: transformation of the wave into a series of discontinuities as it travels creates frequencies that were absent from the signal in the early stages of propagation. Now, higher frequencies tend to dissipate energy faster than lower ones, and this is the reason why shock waves do not appear for moderate signal amplitudes: the temperature rise in the crests is too low to induce a sufficient velocity increase before dissipation flattens the signal.

However this is entirely different, of course, for the pressure levels encountered in nuclear explosions, even at a significant distance from the "zero point". Teller *et al.* provide us a graph of peak stress, or maximum pressure level, for a 100-kt underground nuclear explosion, both experimental and theoretical. The log-log scale with a slope close to -2 is a direct consequence of the conservation of energy: as the shock wave spreads over a sphere of growing radius r, the energy per unit volume in the shock front zone, which is also a pressure, must scale as r^{-2} in order for the energy integral to remain constant over time.

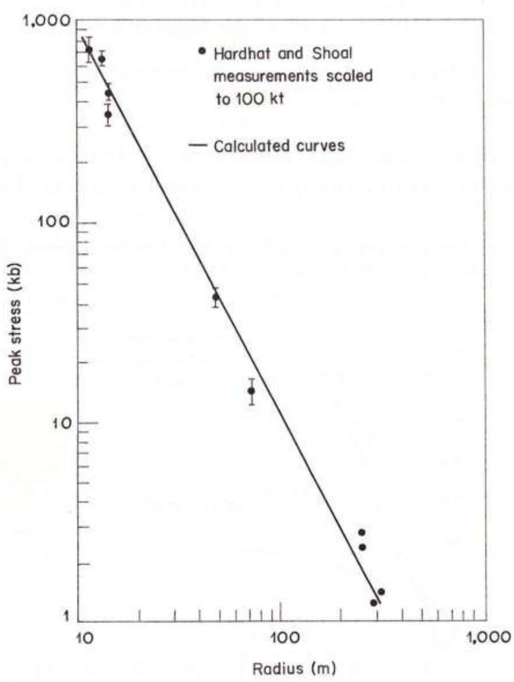

Figure 10: Peak stress (maximal pressure) in kilobars (1 kb = 100 MPa) as a function of distance from "zero point", for a 100-kt underground nuclear explosion, according to Teller *et al.* ([30], p. 132)

Of course, some energy is dissipated since the shock wave irreversibly alters the medium it travels through, but the ability of the rock to dissipate energy is very limited compared to the energy content of the shock wave, that is the reason why it can travel rather far in a quasi-conservative manner.

[23] We take here, for instance, a sinusoidal wave but *not* a shock wave for the moment.

We can see from Fig. 10 that at 300 m from zero point the pressure level is still higher than 1 kb, or 100 MPa; at 100 m, it is around 1 GPa. To give some examples, ultimate tensile strength of granite is not higher than 25 MPa[24] and that of a classical structural steel like ASTM A36, about 550 MPa ([40],[41]) which is a pressure attained at about 150 m from zero point of a 100-kt nuclear explosion in isotropic propagation conditions.

It might look inconsistent to give *tensile* strength of materials while we are dealing with a *compression* wave (which has also some tension counterpart but which figures are not the ones given here); actually this is not the case, and here is why.

We are interested in explosions which occur underground but not far from the surface: deep enough for the explosion to be contained, but shallow enough to cause great damage to superstructures like a high-rise building because of the shock wave, and eventually make it collapse. It is well known that every discontinuity in propagation conditions of a wave generates a reflected wave at the surface where the discontinuity occurs. Coming from underground, a shock wave that reaches the surface meets a condition of zero pressure which generates a reflected wave in phase opposition, *i.e.*, a *tension* (or rarefaction) wave.

Superposition of incident and reflected waves gives, as the incident wave continues to progress, a large *negative* pressure about the same value of the peak stress mentioned above.[25] The result is spallation, as soon as utimate tensile strength of the material lying at the surface is smaller than the peak stress, which is schematically illustrated in Fig. 11. Depending on the explosive yield and on the depth of burial, spallation can occur even on materials which are traditionnally considered as tensile-resistent, such as construction steel.

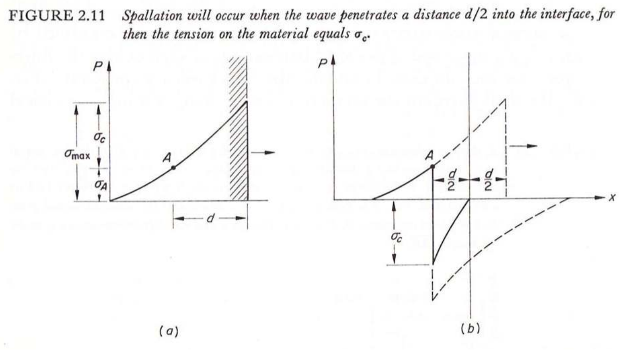

Figure 11: Spallation mechanism occurs near a free surface (zero pressure condition) when the shock wave reaches it. From Teller *et al.* ([30]), p. 68. σ_{max} is the peak stress and σ_c the ultimate tensile strength of the material.

Not only does spallation occur near the surface because of the reflected wave, but closer to the zero point the peak stress is large enough to brake the rock into fine parts even because of the incident compression wave - the finest parts being obviously closer to the center of the explosion. Granite or other hard rocks will therefore turn into a sand-like, brittle material, or into discrete blocks which size will increase with the distance from point zero.

Later, when heat has had time to diffuse through the ground, part of the rock surrounding the explosion cavity will melt and, finally, fall at the bottom of this cavity. Teller *et al.* give, in the case of a 100-kt explosive, a molten layer of about 50 cm thick for a final cavity radius[26] of about 45 m.

[24] Some other sources like [39] give 4 to 5 MPa; granite is not a metrology standard but encompasses a variety of rocks.

[25] This is, by the way, the same mechanism, although with much lower pressure levels, that make a lot of music instruments work - like flutes for instance - and produce a harmonic sound spectrum as a result of multiple reflections on both sides of a 1-dimensional waveguide.

[26] The authors make a distinction between an initial cavity radius and a final one. However, the final radius is achieved in no more than a few hundred milliseconds from time zero.

Derivation of cavity radius

We shall now derive this cavity size as a function of explosion energy and depth of burial, with some degree of empirical adjustment, using experimental evidence from real underground nuclear tests to determine scaling laws prefactors.

Final cavity radius will obviously depend on explosive energy, since converting rock into a high-pressure and high-temperature plasma costs some energy. As the mass, or volume, of rock which is turned into a plasma is proportional to released energy, we expect the cavity radius to scale as the cubic root of the energy:

$$R_c \sim E^{1/3} \tag{56}$$

But as stated earlier, the *final* cavity radius - achieved in less than one second - not only comes from transforming rock into plasma but also from this extremely high pressure plasma expanding and permanently deforming the surrounding rock; we are *not* talking here about the fracturation done by the shock wave but about huge hydrostatic pressure being able to compress the rock. The "final" cavity radius, which may be actually transitory since cavity roof may collapse later, is achieved when plasma pressure equilibrates lithostatic pressure, which depends on depth of burial h through the relation:

$$p_{lith} = \rho\, g\, h \tag{57}$$

where ρ is the rock volumic mass and g gravity acceleration.

During the expansion phase, as stated earlier plasma can be treated as an ideal gas. Furthermore, the speed of this expansion allow us to consider this ideal gas undergoes an adiabatic expansion, and even an isentropic one since almost no dissipative process can take place at this timescale. The Laplace law for an isentropic transformation of an ideal gas between states labeled 1 and 2 reads:

$$p_1 V_1^{\gamma} = p_2 V_2^{\gamma} \tag{58}$$

which can also be written here, since $V = \frac{4}{3}\pi R^3$:

$$p_1 R_1^{3\gamma} = p_2 R_2^{3\gamma} \tag{59}$$

$$\Leftrightarrow \frac{R_2}{R_1} = \left(\frac{p_1}{p_2}\right)^{1/3\gamma} \tag{60}$$

Now we can take for p_1 the plasma pressure at the beginning of its expansion and for p_2 its final pressure which is the lithostatic pressure $p_{lith} = \rho\, g\, h$. It follows that the final radius R_2 will scale as $(\rho\, g\, h)^{-1/3\gamma}$ or, to keep only the variables, as $(\rho h)^{-1/3\gamma}$.

Although values of the heat capacity ratio γ are well known for "ordinary" ideal gases and depend solely on the number of degrees of freedom f of the molecules ($\gamma = 1 + \frac{2}{f}$, which translates in $\gamma = \frac{5}{3}$ for monoatomic gases and $\gamma = \frac{7}{5}$ for diatomic ones at not too high temperatures[27]), it appears that the situation is a little bit more complex here due to the "bimodal" nature of our ideal gas: a material part (nuclei and electrons) and a massless part (photons). According to Teller *et al.* (p. 79), the correct value for a photon gas is $\gamma = \frac{4}{3}$ and, "*purely by chance*", it is also approximately correct for the material part if the temperature is high enough to produce large ionization (but not much greater than 10 000 K, where complete ionization would make γ approach the $\frac{5}{3}$ limit).

As a result, we can write a scaling law for the cavity radius where a constant C remains to be determined experimentally:

$$R_c \approx C\, \frac{E^{1/3}}{(\rho h)^{1/4}} \tag{61}$$

Teller *et al.* give (p. 137) a table of data obtained from 15 underground nuclear explosions; let us cite here only the numerical C values obtained in granite: $C = 57.70$ and $C = 60.48$, for *Hardhat* and *Shoal* tests, respectively.

However, these numbers must be read for R_c expressed in meters, energy expressed in

[27] Otherwise vibrational degree of freedom comes into play.

kilotons of TNT, ρ in g.cm^{-3} (equivalent to 10^3 kg.m^{-3}) and h in meters. Translating this for every variable expressed in SI units (but E in **tera**joules to avoid large numbers), and taking the mean of the 2 values for C (in granite) given by the authors, we can also write the following empirical formula:

$$R_c \approx 206 \frac{E^{1/3}}{(\rho h)^{1/4}} \quad (62)$$

where E is to be expressed in terajoules, ρ in kg.m^{-3}, and h in meters for R_c to be given in meters. For instance, a 80-kt (or ≈ 335 TJ) explosive we estimated in subsection 3.5.3, buried at 100 m depth, would produce a cavity of final radius $R_c \approx 63$ m.

Note however that for a nuclear explosive buried at relatively shallow depth, the lithostatic pressure can vary significantly from the bottom to the top of the cavity, which will account for a non-spherical, but elongated shape of the cavity. The overburden pressure being less important near the surface, expansion of the cavity can be more important in its upper part. We will not develop further than this purely qualitative comment.

4.1.2 Empirical description of phenomena

A number of technical reports on underground nuclear tests are freely available on the internet, sometimes previously confidential but now unclassified, such as "*Some Basic Principles of Scaling Explosion-Produced Damage to Deep Unlined Openings in Rocks*" by G. B. Clark [42], "*Underground Nuclear Explosion Effects in Granite Rock Fracturing*" by S. Derlich (CEA - France) [39], *"The containment of Nuclear Underground Explosions"* by the U. S. Office of Technology Assessment [43], or "*Visual Inspection for CTBT Verification*" by Ward Hawkins and Ken Wohletz [44]. Some more recent scientific papers can also be found easily (see for instance [45]).

All these documents give the same overall picture of underground effects of nuclear explosions, but with numerical values that can vary significantly because of the different geological nature of the test sites and of the inherently fuzzy definitions of the different damaged zones considered.

Let us first read some qualitative description of the phenomena that occur just after a deep underground nuclear explosion (*i.e.* an explosion that does not produce ejections of solid matter in the atmosphere), as they are exposed in *"The Effects of Nuclear Weapons"* [36] on pages 61-62.[28] The following lines are excerpts from sub-sections 2.101, 2.102 and 2.103. Emphasis is added to separate the four phases.

"*The phenomena of deep underground detonations can be described best in terms of four phases having markedly different time scales.*

First, the explosion energy is released in less than one microsecond. *As a result, the pressure in the hot gas bubble formed will rise to several million atmospheres and the temperature will reach about a million degrees within a few microseconds.*

In the second (hydrodynamic) stage, which generally is of a few tens of a second duration, *the high pressure of the hot gases initiates a strong shock wave which breaks away and expands in all directions with a velocity equal to or greater than the speed of sound in the rock medium. During the hydrodynamic phase, the hot gases continue to expand, although more slowly than initially, and form a cavity of substantial size. At the end of this phase the cavity will have attained its maximum diameter and its walls will be lined with molten rock. The shock wave will have reached a distance of some hundreds of feet*[29] *ahead of the cavity and it will have crushed or fractured much of the rock in the region it has traversed.*

[28] A similar description can also be found in [43], Chapter 3, p. 32, in [34], p. 35, or in [30], Chapter 4, p.133.

[29] 100 ft. $\approx$ 30,5 m

The shock wave will continue to expand and decrease in strength eventually becoming the "head" (or leading) wave of a train of seismic waves.

During the third stage, *the cavity will cool and the molten rock material will collect and solidify at the bottom of the cavity.*

Finally, the gas pressure in the cavity decreases to the point when it can no longer support the overburden. *Then, in a matter of seconds to hours, the roof falls in and this is followed by progressive collapse of the overlying rocks. A tall cylinder, commonly referred to as a "chimney," filled with broken rock or rubble is thus formed. If the top of the chimney does not reach the ground surface, an empty space, roughly equivalent to the cavity voIume, will remain at the top of the chimney. However, if the collapse of the chimney material should reach the surface,* ***the ground will sink into to the empty space thereby forming a subsidence crater.*** *The collapse of the roof and the formation of the chimney represented the fourth (and last) phase of the underground explosion."*

Although [36] gives a schematic picture of the resulting zones underground (Fig. 2.103 p. 62), we prefer to show here (Fig. 12) another illustration found in a paper originating from French *Commissariat à l'Énergie Atomique* (CEA) [39] which we think to be more precise. It introduces several radii (R_c, R_b and R_f) measured from the shot point that will be discussed later. An even more realistic picture, describing a real case of deep underground explosion in former USSR ("borehole 102" experiment, Balapan test site), can be found in [34], p. 12, and we give it in Fig. 13 where a spall zone is clearly identified near the surface (see 4.1.1 for explanations).

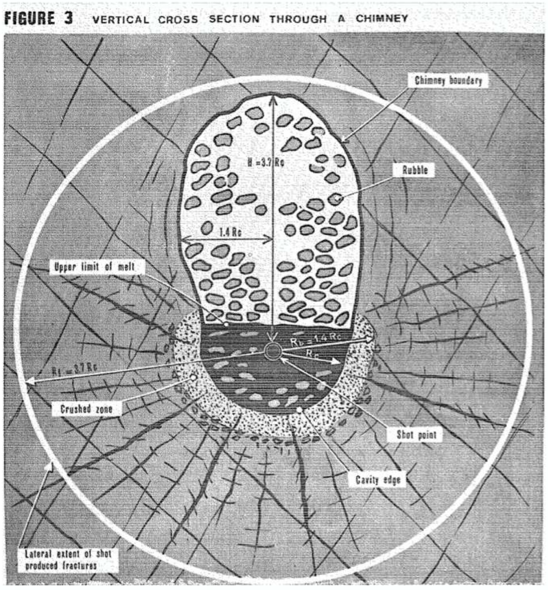

Figure 12: Vertical cross section through a chimney resulting from a deep underground nuclear explosion. From [39].

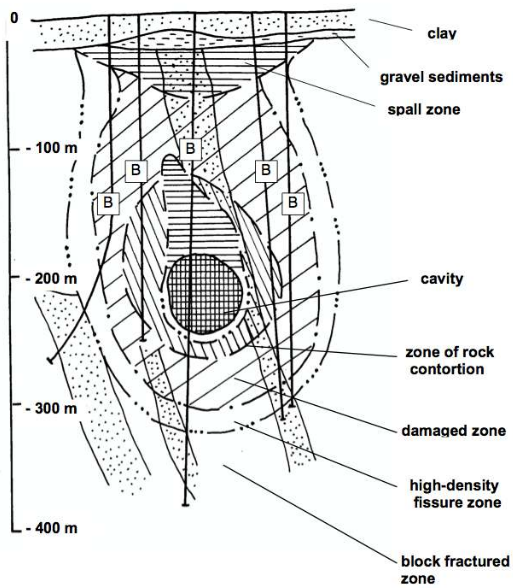

Figure 13: Same kind of diagram as Fig. 12 but for a real nuclear test in former USSR (Balapan test site). From [34], p. 12, Fig. 4.

Cavity radius data

We report in table 1 some empirical scaling laws provided by the CEA paper [39], with W the yield in kilotons and the radii

$R_c = 7.3\,W^{1/3}$	cavity radius
$R_b = 10\,W^{1/3}$	crushed zone radius
$R_f = 26\,W^{1/3}$	fractured zone radius
$R_r = 35\,W^{1/3}$	stressed zone radius

Table 1: Different radii as defined in Fig. 12 as a function of bomb yield expressed in kilotons TNT equivalent.

$R_c = 4.5\,E^{1/3}$	cavity radius
$R_b = 6.2\,E^{1/3}$	crushed zone radius
$R_f = 16\,E^{1/3}$	fractured zone radius
$R_r = 22\,E^{1/3}$	stressed zone radius

Table 2: Same as table 1 but with bomb energy expressed in terajoules.

expressed in meters, for tests conducted in granitic batholith in the Sahara desert near the Hoggar mountains, with about 1000 m of overburden pressure.

As 1 kiloton TNT equivalent equals 4.184 TJ we can also reformulate these laws in table 2 using the energy E released by the bomb in terajoules.

Note that all these values are approximate and depend heavily on the boundaries definitions. According to [39], "*crushed zone is encountered from 7.3 to 10 $W^{1/3}$*." If we compare the cavity radius scaling found here with that given by Teller *et al.*, taking into account $h = 1000\,\text{m}$ and $\rho = 2.63\times10^3\,\text{kg.m}^{-3}$, we find that Equ. 62 gives $R_c \approx 5.1\,E^{1/3}$, quite close to the CEA value.

Other authors like Fokin [46] or Rogers [47] include in their scaling laws the dependence on depth of burial and rock volumic mass we derived above.

Rogers gives the following relation, identical to that proposed by Teller *et al.*, for the final cavity radius (in meters):

$$R_c = C\,\frac{W^{1/3}}{(\rho h)^{1/4}} \qquad (63)$$

where $C \approx 59$ for granite (from 57 to 61), when W is in kilotons, h in meters and ρ in $g.cm^{-3}$.

On the Russian side, Fokin gives three relations, some with detailed parameters like the initial value of the adiabatic exponent, the adiabatic exponent for the equilibrium part of the detonation products and the volumic mass of the explosives; and finally gives the following approximated relation:

$$R_c = 0.2842\left(\frac{E_0}{P_h}\right)^{1/3} \qquad (64)$$

where E_0 is the energy expressed in $kgf.m$ and P_h is the counterpressure at the depth of explosion defined as:

$$P_h = \sigma_{comp} + \rho h$$

with σ_{comp} the maximum strength[30] of the rock under compression expressed in $kgf.m^{-2}$ and ρ the volumic mass[31] of the rock in $kg.m^{-3}$. Translating Equ. 64 in SI units (pascal for stress, joule for energy) we get the same formula since the numerical value of $\frac{E_0}{P_h}$ does not change.

Note that for shallow depths of burial $\rho h \ll \sigma_{comp}$, so that the radius can be considered independent of h. In order to achieve $\rho h < 0.01\,\sigma_{comp}$ we must restrict for granite, taking $\sigma_{comp} = 200\,\text{Mpa}$, to $h \lesssim 740\,m$. Note also that formulas 63 and 64 seem to be incompatible since the former does not take into account the compressive strength of the rock, whereas the latter does. Moreover, Equ. 63 scales as $h^{-1/4}$ whereas Equ. 64 is practically independent of h in the shallow depth approximation but scales as $h^{-1/3}$ in the opposite case. And expressions given in 1 do not even take into account depth of burial whereas 63 and 64 do, because they are restricted to some narrow depth of burial range. Even this is not satisfying from a theoretical point of view, it should be kept in mind that all these expressions are approximate laws based on experimental values collected after nuclear tests, and that if

[30] Compressive strength for granite can vary between about 100 and 300 MPa [40].

[31] Volumic mass for granite is about $2.7 \times 10^3\,kg.m^{-3}$.

Authors	Cavity radius (m)
CEA [39]	34
Teller *et al.*[30]	38
Rogers [47]	38
Fokin [46]	36

Table 3: Some values of final cavity radius for a 100-kt bomb in granite at 1000 m depth (taking $\rho = 2.63 \times 10^3\,\mathrm{kg.m^{-3}}$), according to different authors.

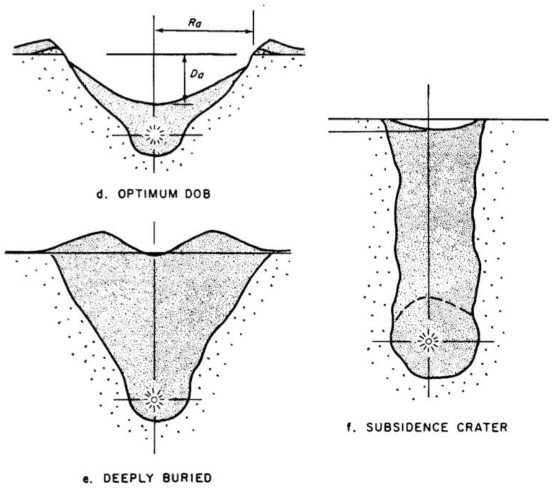

Figure 14: Different possible effects of underground nuclear explosions depending on depth of burial. From [36], part of Fig. 6.06, p. 234. R_a and D_a are the apparent radius and depth, respectively. DOB = Depth Of Burial.

other factors play only a minimal role, some authors may choose to use simplified laws where only energy determines the cavity radius.

Les us calculate how numerical results compare for these different expressions, in the case of a 100-kt (418 TJ) explosion at a 1000 m depth in granite, and taking for the Fokin relation $\sigma_{comp} = 200\,\mathrm{Mpa}$ which is the value given by the CEA study. We summarize in Table 3 the final radius cavity obtained from table 1 and expressions 61, 63 and 64.

Note that in this depth range the relation given by Fokin gives almost no dependance on h since $\rho h = 2.63 \times 10^6\,\mathrm{Pa} \ll \sigma_{comp} = 2 \times 10^8\,\mathrm{Pa}$. As we can see, although relations given by different authors may differ formally and even look theoretically incompatible, as a practical tool they manage to give similar results, provided they are used in the case they are designed for.

The above example addresses the case of a deeply buried explosive; however we are more interested in explosions which, although still contained, will produce some striking effects at the surface.

On Fig. 14 we give an overall picture of different explosion effects depending on the depth of burial of the device, as shown in [36] p. 234. These three cases do not include very deep underground explosions where no permanent deformation of the ground surface to the vertical of the device, around the so-called "*ground zero*" or "*surfaced ground zero*" point, occurs.

In case *f* of Fig. 14, the initial cavity created by the explosion finally creates a rubble chimney between the shot point and the surface, once the pressure inside the cavity has decreased enough and no longer supports fractured rock above the cavity. Let us emphasize again that fracturation of the rock occurs just after the explosion - before the expansion of the cavity - due to the enormous pressure generated by the shock wave.

Moreover, because of the almost discontinuous, extremely abrupt change in pressure generated by the shock wave - much more than for chemical explosions - it can be inferred that there will be very little difference in behaviour of materials that will be in the shockwave path: even ductile materials like steel can in these particular conditions appear as brittle. The ductile-brittle transition in metals is generally considered as a function of temperature[32], since low temperatures slow down

[32] in the same way as glass transition in polymers

dislocations movements that dissipate energy. But it can be also regarded as a transition depending on deformation speed, just as the time-temperature superposition principle [48] suggests for "soft materials" physics: decreasing temperature is equivalent to increasing deformation speed.

4.2 Proposed mechanism

We have so far demonstrated that the only known source of energy that can account for the huge amount of heat ($\sim$ 1 PJ) released from Ground Zero remnants during the months following September 11, 2001 terrorist attacks was nuclear, and have given some basic review of known effects of underground nuclear explosions. In the following, we will see how a nuclear demolition device could have been rationally designed for skyscrapers and implemented for some of them, without any terrorist intentions but based solely on efficiency and cost criteria. Any discussion about the opportunistic use of such devices for - or during - terrorists attacks, and about the identity of the people deciding it or their motives, is outside the field of physics and will be left up to the readers discretion.

4.2.1 General mechanism

As stated above, an underground nuclear explosion creates a tremendous shock wave that is able to shatter every material and turn it into tiny pieces or larger chunks depending on the distance from point zero, up to some limit where the wave will have only elastic and non-destructive behaviour. **This is the main feature that can explain why it can be used to collapse buildings, even steel-framed ones like skyscrapers**, provided that the nuclear explosive is placed at the right depth underneath the building **in order to create a subsidence crater at ground level that triggers the collapse**. An example of this rather counter-intuitive (a depression at ground surface resulting from extremely high pressures underground) but very well known phenomenon is given on Fig. 15 and various videos of it are available on the Internet, see for instance [49] on AtomCentral YouTube channel.

Figure 15: Numerous examples of subsidence craters created by underground nuclear explosions can be found for instance at Nevada test site in the USA. Although this one is in tuf, hard rocks such as granite can also produce a similar, counter-intuitive phenomenon.

Release of a large quantity of heat is only a necessary side-effect, as well as release of some radioactive elements. Because of energy conservation, heat release cannot be avoided - it is directly proportional to explosive energy or "yield" - but is only delayed due to high thermal resistance of surrounding rock; however radioactive contamination, although unavoidable, can be restricted to some safe levels according to Teller *et al.*[33], provided some precautionary measures

[33] More specifically, according to them "*The fusion explosions* [...] *can be handled in such a way as to eliminate most of the ensuing residual radioactivity.*" (p. 3). Note also that although the Chagan nuclear test performed at the Semipalatinsk test site on January 15, 1965 used a fission device, the resulting lake was declared safe by the soviet authorities and a small movie [50] even showed some swimmers dipping into it shortly after, wearing only a small swimsuit and using only a simple snorkel.

are taken. The aim of an engineer for designing such a demolition device must therefore be to maximize the desired effect (collapsing the building) and at the same time to minimize the undesirable effect (radioactive fallout). A compromise has to be found between a zero point situated very deep underground, in order to contain radioactive elements as much as possible, and a shallower one, to maximize shock wave shattering.

If the purpose is to destroy a high-rise building, using an underground explosive will necessarily shatter more effectively the lower part than the upper part of the building, since the shock wave maximum pressure decreases approximately as the inverse square of distance from point zero, as seen above (subsection 4.1.1 and Fig. 10). However, to some extent, the structure of the building itself might act as a "waveguide" and produce a peak stress at the top higher than it would be for the same traveling distance in a homogeneous and isotropic material; furthermore, as explained above, reflection of the wave at the top produces a tension wave which negative pressure is more effective at breaking materials.

Apart from the shock wave effect itself, a very important feature of a nuclear underground explosion is, in most cases, *the creation of a rubble cylindrical chimney as a consequence of the cavity "roof" collapse*. As a consequence, *there is no need for the initial cavity to reach the basement of the building to make it collapse*: it is enough to make the upper part of the rubble chimney reach the basement. In this case, the building having been - at least in its lower part - shattered by the shock wave but kept apparently unchanged in a kind of "metastable" state, it will finally collapse when its basement no longer finds mechanical support - or a much weaker one - because of the rubble collapsing into the cavity. According to Teller *et al.* (Fig. 4.8 p. 138), the ratio of the chimney height[34] to the cavity radius is around 4.35, an experimental mean value obtained from several shots in granite. If we consider, in the case of the World Trade Center, an estimated energy of 80 kt TNT equivalent[35] or $\approx \frac{1}{3} \times 10^{15}$ J per explosive as we mentioned earlier in subsection 3.5.3, we can make several hypothesis for their depth of burial and see which one fulfills the best an engineers team specifications for building controlled demolition.

1. Let us first suppose the explosive is buried 100 m below the ground level. The radius of the cavity will be, according to Equ. 62 and taking $\rho \approx 2.7 \times 10^3\,\mathrm{kg.m^{-3}}$ for the rock volumic mass:

$$R_c \approx 206 \frac{\left(\frac{1}{3}10^3\right)^{1/3}}{\left(2.7 \times 10^3 \times 100\right)^{1/4}} \approx 63\,\mathrm{m} \quad (65)$$

   As said before, in this case the dimension of the cavity itself is on the same order of magnitude as the depth of burial, so that lithostatic pressure cannot be considered the same at the top and at the bottom of the cavity and the cavity itself will not grow to a spherical shape, even before collapse of the fractured rock leading to the rubble chimney. This calculation should therefore be considered as a very crude one and the "radius" of about 60 m only taken as a rough guide.

   In this case, the theoretical chimney height being $h \approx 63 \times 4.35 \approx 275\,\mathrm{m}$, it is clear that ground surface - and the building itself - stops chimney growth so that the building basement falls into a local depression, which is a desired

[34] Defined as the distance between zero point and the chimney top.

[35] Let us recall here that we estimated this energy only through heat released by free convection at Ground Zero, and that it should therefore be under-estimated since free convection is not the only heat transfer mechanism involved, and since explosive energy is not entirely released as heat. However, we choose here to use this "conservative" value.

effect that can trigger building final collapse. However, it can be argued that the distance between the cavity top (before chimney formation) and the buiding basement, about 40 m, could be insufficient to satisfactorily contain radioactive elements that can migrate from the cavity to the surface through the rubble. In other words, from the radioactive pollution criterion it should be safer to chose the maximum depth that still can trigger building collapse, that is, a depth for which only the top of the rubble chimney reaches the building basement.

2. Let us then try this case, and choose a depth of burial of 200 m below ground surface. Now the cavity radius is slightly reduced:

$$R_c \approx 206 \frac{\left(\frac{1}{3}10^3\right)^{1/3}}{(2.7 \times 10^3 \times 200)^{1/4}} \approx 53\,\mathrm{m} \quad (66)$$

Now the chimney height is about $h \approx 230\,\mathrm{m}$, which is enough to make sure the basement will fall into a depression to trigger the building collapse, but the distance between the initial cavity top and the building basement is a more secure 150 m. But another problem now emerges: the distance between the "zero point" and the top of the building might be too large in order to weaken its structure sufficiently. In the case of the Twin Towers, distance varies from 200 m in the basement to more than 600 m at the top. As can be inferred from Fig. 10 taken out Teller *et al.*, peak stress for a 100-kt explosive[36] at 600 m distance - in a homogeneous and isotropic medium - will be in the 350 bar (or 35 MPa) range, and as seen in paragraph 4.1.1 this value is larger than ultimate tensile strength of granite (maximum of 25 MPa) but smaller than that of common structural steel (550 MPa). At 200 m distance, it will be roughly 300 MPa. Although real strain on structural materials would require much more precise analysis, this order of magnitude calculation already tells building destruction is no more certain, or at least that the upper part of the building is likely to remain unshattered.

We face here a typical optimisation problem where two effects - a desired one and an undesirable one - pull in opposite directions and a compromise has to be chosen which depends on the weighting of the two; engineers must solve such problems all the time for cost, safety or regulations reasons. At this point it is impossible to give any precise estimate of the optimum depth of burial of a nuclear demolition device, since we do not know its precise energy content (or "yield") nor which minimum depth can be considered - or was considered, in the 1960's and in the USA - as "safe" for radioactive pollution issues (see Appendix B). Order of magnitude arguments have their virtues - we hope to have demonstrated this - but cannot replace more precise and tedious calculations when fine tuning is sought after.

We therefore conclude that a nuclear explosive, *on the order of* 100 kt TNT equivalent in energy and buried at a depth *on the order of* 100 m - and this should *really* be taken as mere powers of ten estimates - can destroy a high-rise building using *both* the shock wave created by the explosion *and* the subsidence crater produced by the collapse chimney created underground, once the cavity pressure has decreased to a level where broken rock can no longer sustain overburden. As an illustration, we draw in Fig. 16 the approximate position of a nuclear explosive relative

[36] We choose here a 80-kt explosive for our calculations although, as said above, the exact value might well be larger and even larger than 100 kt. But anyway, it should be kept in mind that initial temperature and pressure values derived from Equ. 50 and 53 come from an energy *volumic density* estimate, and not from an energy estimate. If the 80-kt has the same energy per unit volume as the 100-kt one, calculations remain unchanged.

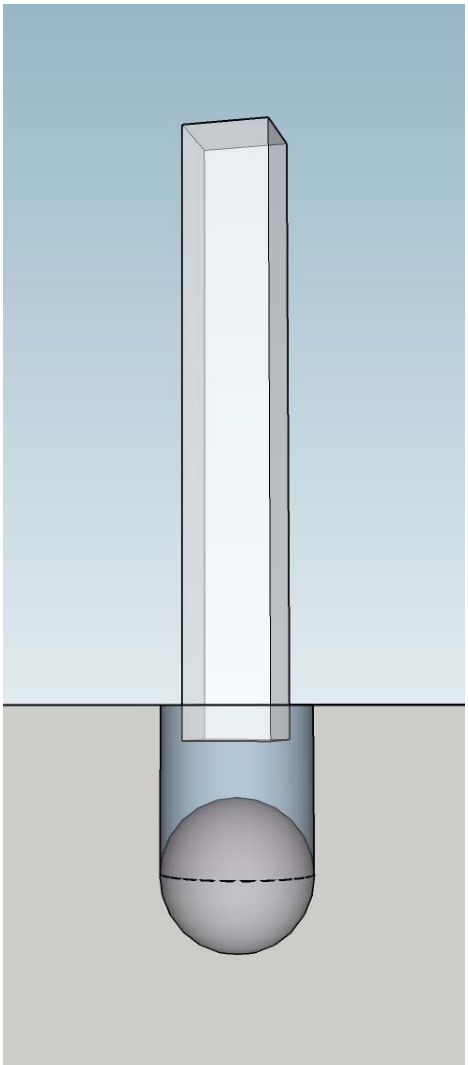

Figure 16: Possible position of a 80-kt nuclear explosive (at center of sphere), initial cavity (sphere) and approximate boundary of rubble chimney (cylinder) for nuclear demolition of WTC1 or WTC2, drawn to scale with the building and its basement. Here a depth of 120 m under ground surface has been chosen. Collapse chimney may have not exactly the same diameter as cavity, which would in this case adopt a vertically elongated shape (not displayed) because of lithostatic pressure differences from top to bottom.

to WTC1 or WTC2 buildings, as well as approximate dimensions of initial cavity and rubble chimney. Note that the cavity - hence the chimney - diameter is larger than the building footprint (a square of 64 m width, which has a 90m diagonal).

Finally, we would like to emphasize again the very large time scale difference between the first effects of the explosion and the last mentioned by the different authors (collapse of the "roof" and chimney formation), which are actually not the last ones since the enormous amount of energy generated converts mainly into heat and heat diffusion through the surrounding medium will take place on a much larger time scale, even if the chimney top is close to the ground surface. This characteristic cooling time will be similar to the ones encountered for thick lava flows after volcanic eruptions, *i.e.*, several months, which corresponds to the timescale we estimated in Equ. 23. In a 1972 paper [51], the French Commissariat à l'Énergie Atomique investigated the temperature distribution in the rock 178 and 221 days after a fully contained nuclear test in granite. It turned out that the maximum temperatures measured, in the vicinity of the shot point, were respectively about 600°C (for 178 days $\approx 1.5 \times 10^7$s) and 500°C (for 221 days $\approx 1.9 \times 10^7$s). This is entirely consistent with the very long characteristic time of $\tau \approx 10^7$ s (Equ. 23) we have chosen for estimating the total amount of heat released at Ground Zero.

4.2.2 Differences between WTC1, WTC2 and WTC7

There have been obvious differences in WTC1 and WTC2 collapses, on one side, and WTC7 collapse, on the other side. Not only because the Twin Towers collapsed in the morning of September 11, 2001 (at 9:59 am and 10:28 am EDT[37], respectively) after having allegedly been struck by airlin-

[37] Eastern Daylight Time, or UTC-4.

ers, whereas WTC7 collapsed only in the afternoon of the same day (at 5:20 pm EDT) without having been hit by any plane, but also because the last collapse appeared to many building demolition experts as an example of a properly made controlled demolition, whereas the first two were rather catastrophic events from this perspective - even if we do not take into account the tragic fact that many people died in the process.

Such a difference needs to be explained, at least qualitatively, since we claim that the demolition method employed was the same. The first obvious difference between the Twin Towers and WTC7 resides in their height: according to NIST in document NCSTAR 1-1 ([52]), p.7, WTC1 was 1368 ft (417 m) high[38], not including the antenna, and WTC2 about the same (1362 ft or 415 m), both consisting of square structures with a side dimension of 207 ft (63 m), whereas the overall dimensions of WTC7 ([52], p. 13) were approximately 330 ft (100.6 m) long, 140 ft (42.7 m) wide, and 610 ft (186 m) high.

The Twin Towers were then more than twice as tall as WTC7, which makes a huge difference if a demolition process uses a powerful underground explosive designed to shatter the building structure before making it collapse thanks to an underground depression. It appears from infrared imaging that thermal energy released at the three building locations was roughly the same, at least on an order of magnitude level, therefore shock wave pressure levels must have been also similar. Since they decrease as a function of height in the building structure, it turns out that the much smaller WTC7 could have its structure much more effectively shattered than those of WTC1 and WTC2, which corresponds to observation: both WTC1 and WTC2 collapsed with an upper part falling apparently as a single entity.

Furthermore, the structure of WTC7 was very different from that of the Twin Towers.

[38] above the Concourse level

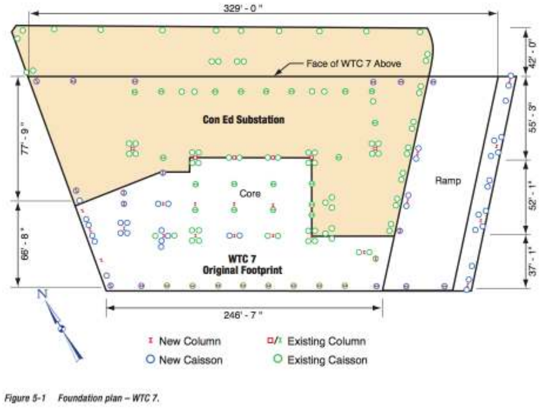

Figure 17: Fig. 5-1, p. 5-3 of [53], shows Con Edison power substation footprint, which differs from that of WTC7.

Still according to NIST in document NCSTAR 1A ([25]), p. 5, in WTC7 "*the layout of the columns did not align with the building foundation and the Con Edison columns*[39]. *Therefore, a set of column transfers were constructed within the volume bounded by the 5th and 7th floor slabs*". In short, the building had a larger footprint than its foundations, and up to the 7th floor exterior columns had no structural role but could rather be seen as "hanging" from the upper part rather than sustaining it. The Federal Emergency Management Agency gives in a 2002 document [53] the foundation plan of WTC7 that we reproduce in Fig. 17.

According to [53] again (p. 5-4), "*An interior braced core extended from the foundation to the 7th floor. The horizontal shear was transferred into the core at the 5th and the 7th floors.*". So, if a shock wave was transmitted to the building from a deep underground located explosive, it was necessarily through this core only and not through the entire footprint of the building. As a consequence, exterior columns up to the 7th floor experienced no or little shattering from this shock wave, and exterior walls up to

[39] WTC7 was built on top of an existing Con Edison electric power substation. See for instance [54].

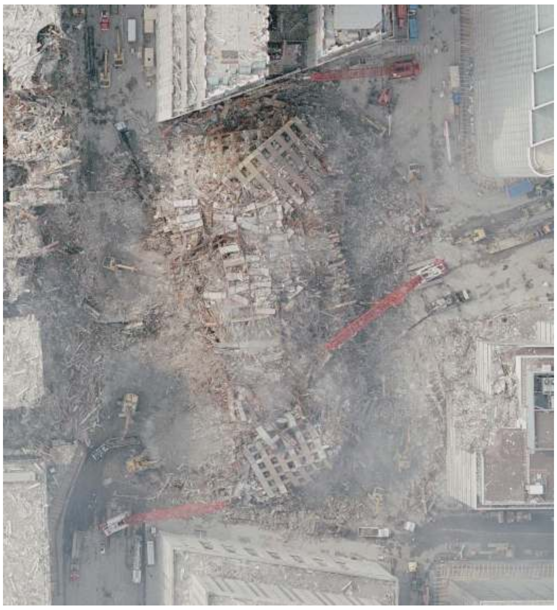

Figure 18: Aerial picture of WTC7 rubble pile during cleanup operation, cropped from [55]. Some almost undamaged facade walls parts are clearly visible, covering some much more shattered debris.

this height could not be destroyed. This is consistent with what was observed as numerous photographs of the WTC7 ruins can prove; see for instance Fig. 18 from National Oceanic and Atmospheric Administration [55].

We conclude about WTC7 that both its much lower height and its very specific structure can easily explain the observed differences between its collapse and those of WTC1 and WTC2, assuming the same kind of underground nuclear explosive was used for its demolition.

5 Open questions

We have so far demonstrated that the energy source responsible for the huge amount of heat released at Ground Zero after the September 11, 2001 terrorists attacks was necessarily nuclear, and that basic knowledge about underground nuclear explosions could give us a sensible demolition mechanism for the WTC1, WTC2 and WTC7 buildings, consistent with the very common idea - in the 1960s - that nuclear explosives could be used not only for military purposes, but also as a peaceful and powerful engineering tool.

In the following, we will make no demonstration but rather suggest our colleagues to investigate secondary issues that may probably be related to the use of underground nuclear explosives. However, having made no detailed investigations by ourselves on the following questions, some of them may have little or even no relevance to the main subject of our paper; we welcome in any case all efforts to clarify these items.

5.1 Accidental destruction of WTC6

The WTC1, WTC2 and WTC7 collapses were spectacular events by themselves, but some neighbouring buildings also suffered some extremely unusual damages, especially WTC6 which was located between WTC1 and WTC7. Numerous aerial pictures show very large, round holes on top of the building, but that appear to go very deep inside up to ground level or even deeper. Although they are generally attributed to very large parts of WTC1 falling on top of WTC6, we suggest that another explanation could be investigated: namely, that some unwanted underground collapse mechanism could have partially destroyed WTC6 in just the same way it destroyed WTC1, the foundations losing support as a side-effect of WTC1 cavity formation.

5.2 Surprisingly low debris pile

Many observers noted that the debris piles for the 3 buildings which collapsed on September 11, 2001 in New York City were

Figure 19: On this aerial photograph of World Trade Center, WTC6 building exhibits very large and deep holes (cropped from [55]).

surprisingly small compared to their standing size. We have not investigated this question, but if the observation is exact, it could be explained precisely by the subsidence crater that is formed after an underground nuclear explosion occurs.

Similarly, it could be interesting to study LIDAR pictures taken at Ground Zero after the attacks in order to detect possible depressions originating in underground nuclear explosions. Such pictures have widely circulated, one of them even being the cover picture of a book: "American Ground - Unbuilding the World Trade Center" by William Langewiesche, already cited here [11]. LIDAR being commonly used in geodesy and seismology (among other uses) to produce accurate measurements of altitude, it should be possible to know if ground level around WTC1, WTC2 and WTC7 exhibited some anomalies that could be attributed to collapse chimney formation.

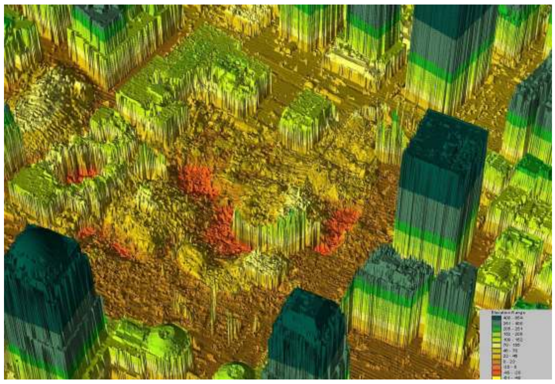

Figure 20: One of numerous LIDAR pictures that were made of Ground Zero. Orange levels, around WTC1 and WTC2 ruins and in the WTC6 "big hole" on the left, are negative elevations (0 to -40 feet) relative to street level. (EarthData, September 17, 2001. Photo by NYC Office of Emergency Management/Getty Images).

5.3 "Bathtub" partially destroyed

If a building is to be destroyed thanks to a shock wave coming from deep underground, one must expect underground structures to be severely damaged, whereas if destruction is achieved only thanks to aerial explosions - as it is the case with conventional controlled demolition - no aftermath is to be expected underground. Precisely, in the case of the World Trade Center we have some evidence that the slurry wall around the complex, the so-called "bathtub"[40], was severely damaged [56] at least near WTC2 south-west facade, although it was not directly in the footprint of any of the three collapsed buildings. Furthermore, there is also some evidence [57] that the basement levels of the Twin Towers were totally destroyed, which would be impossible if, as it is often said, collapse had been triggered by thermal weakening of the structures, *but also* if conventional con-

[40] which was built in order to prevent water from Hudson River, permeating through the soil, to flood underground levels.

trolled demolition methods had been used.

These evidences for severe damage, if not total destruction of some of the underground structures of the World Trade Center imply that some destructive device was necessarily located underground, since a building collapse per itself can not produce such an effect.

5.4 Seismic signals of characteristic type

As it is the case for any kind of powerful underground explosion, detonating a nuclear underground explosive produces a seismic event, the magnitude of which depends on the energy or "yield" of the explosive. As seismic signals can travel very large distances, and be detected thousands of kilometers away from the explosion location when the explosive event is large enough, seismic signals have been used for decades for detecting unclaimed underground nuclear tests since the Treaty on the Non-Proliferation of Nuclear Weapons entered into force in 1970. Naturally occuring seismic events differ markedly from artificial ones, be they caused by chemical explosives - in the case of mining activity for instance - or nuclear ones.

Some authors have tried to give a relation beween the explosive "yield" (energy) and the seismic magnitude which results from the explosion. For instance Teller *et al.* [30] propose in *The constructive Uses of Nuclear Explosives* (p. 300):

$$M = 3.64 + \log_{10} Y \qquad (67)$$

where Y is the yield in kilotons and M the magnitude from a Wood-Anderson torsional seismograph. Applying this relation to a 80-kt nuclear device would give $M \approx 5.5$. Other authors like Argo *et al.* [58] give the following rule:

> "*A 1 kiloton (kt) explosion close-coupled in hard rock is roughly equivalent to a seismic magnitude of m_b 4.0: a 1 kt explosion decoupled (that is fired in a large cavity) is equivalent to around m_b 2.0.*"

It appears that formula 67 gives $M = 3.64$ for a 1-kt nuclear device, which is consistent with above rule. However Argo *et al.* let a much wider range open, since a difference of 2 units in magnitude, which is a logarithmic scale, is indeed a very large one.

There has been a comprehensive work on seismic waves recorded on September 11, 2001 by Kim *et al.* [59]. However, this paper gives a maximum seismic magnitude (M_L) of 2.3 for WTC1 collapse, of 2.1 for WTC2 collapse and *does not address* the case of WTC7 collapse, although some minor but significant signals ($M_L = 0.9$ and $M_L = 0.7$, in chronological order) were recorded for what authors say to be airplane impacts. These 2.1 and 2.3 magnitudes, even in the high decoupling hypothesis, is clearly inconsistent with the underground detonation of a 80-kt nuclear device, or even a 50-kt one.

We consider so far this paper as inconsistent with our findings, and would appreciate comments and further work by seismology specialists, especially regarding WTC7 collapse records. Despite some efforts, we did not manage to get several independent scientific papers addressing seismic waves generated by September 11, 2001 events in New York City, and as science needs always to be reproducible, we consider this situation as unsatisfactory.

Moreover, we consider that Kim *et al.* paper raises some questions for which we have no answer; for instance, even with modest magnitudes of $M_L = 0.9$ and $M_L = 0.7$, the so-called "plane impacts" seem to defy the laws of physics, since it seems extremely unlikely that even a big airliner hitting horizontally at full speed[41] a skyscraper can give rise to

[41] Some observers, especially professional pilots, have noticed [60] that full speed was not attainable at sea level in any case for these planes.

any detectable seismic signals. The only effect a physicist would expect is a vibration motion of the buildings, which are basically cantilever beams, at their natural frequency.

5.5 White fumes caused by water evaporation/condensation

As it has been said before, fires lasted for more than three months at Ground Zero, which in itself deserves an explanation. But what appeared to emerge from Ground Zero into the atmosphere were not mainly fire smokes, which are usually rather black especially in the case of oxygen-starved fires, but white fumes which looked very much like clouds of water condensation. This is exactly what is to be expected from a zone where coexist a huge heat source and large amounts of water in the soil, coming from Hudson River, especially once the "bathtub" was partially destroyed. In order for fumes to be white, small droplets or solid parts that constitute it must be transparent, and therefore it excludes carbon-rich particles which are generally found in fires aerosols, which are black.

Geysers springing from the street

Even more conclusive, some large geysers springing from the ground level have been recorded on video during the collapse of WTC2 ([61] at 9:51). This observation, which is strange if one accepts the idea of a "natural" collapse or even that of a conventional controlled demolition, becomes much less mysterious if one knows that an extremely large heat source resides underground, and that water is present both because of soil humidity and of New York City underground water supply system. It is therefore not surprising that some accidental contact between extremely hot temperature material and water can produce a powerful vapor eruption.

We believe this aspect should equally be investigated, but it is not the subject of our work.

5.6 Extremely large amount of dust, that was *not* mainly concrete dust but also steel dust

A striking fact about the collapses that occurred at the World Trade Center was the extremely large amount of dust generated, which particles were particularly small and consequently deposited onto the streets only very slowly. It was also a rather dark dust, as numerous photographs have pictured it, and a short "night" was observed at street level in NYC just after WTC1 and WTC2 collapses, even they did occur in the morning of a sunny day.

Apart from foundations, concrete was a minor component of the three skyscrapers; lightweight concrete was used in WTC1 and WTC2 as a 10 cm thick slab (see NIST report [24] p. 10) at each floor, mainly for acoustic reasons.

Although producing steel dust usually involves abrasion rather than explosions, we suggest that the extremely sharp wavefront generated by a nuclear explosion, compared to a chemical one, could produce on steel some effects that are usually encountered on fragile materials like concrete. A frequency analysis (Fourier transform) of a nuclear shockwave will contain much higher frequencies that that of a chemical explosion shockwave. Given that submitting materials to higher frequencies is generally equivalent to lowering temperature (something well known in the field of polymer physics as time-temperature superposition principle), we suggest that the possibility for a nuclear underground explosion to produce fine particles of steel because of the shockwave should be investigated.

5.7 Extremely rapidly corroding steel remains

Number of observers have reported an extremely fast corrosion process occuring at the surface of steel remnants at Ground

Figure 21: Some orange fumes observed at Ground Zero, from [63]

Zero. It is of course not surprising that unprotected steel rusts over time, especially in a moisty and hot environment which was the case here. However, the pace of the process was described as surprising, and may be explained by a side effect of a nuclear explosion, namely gamma rays production. It is known for long that gamma irradiation in an air/water environment can produce nitric acid because of radiolysis of nitrogen gas and water; see for instance Etoh *et al.* [62].

There were huge amounts of water present at Ground Zero after the attacks, both because of firefighters action and because of a partial destruction of the "bathtub", or slurry wall, which is in itself a significant event as explained above. If a large amount of gamma radiation was produced because of a nuclear explosion, a subsequent production of nitric acid was unavoidable. As nitric acid is a known factor of rapid corrosion for structural steels, we suggest that fast corrosion could be a side effect of nuclear reactions that occurred on the site. Furthermore, nitric acid fumes are known to be orange, and this was precisely the colour of some fumes that were observed at Ground Zero in the first days after the attacks. Colour itself is no proof of nitric acid presence, of course, so we consider this idea as an open question.

5.8 Tritiated water

Traces of tritium, in the form of tritiated water, (HTO) have been found at Ground Zero [14] although this particular chemical element is usually considered as a signature of the occurence of a nuclear reaction, and in any case the result of human activity. Ccosmic rays do produce tritium when interacting with atmospheric gases, but this effect is not enough to explain what was found at Ground Zero.

The possible sources for tritiated water was, according to authors, tritium radioluminescent devices (emergency signs) that were supposed to be present in the 2 planes that presumably hit the Twin Towers (WTC1 and WTC2), watches belonging to the airplanes passengers and possibly, weapons belonging to federal and law-enforcement agencies, accidentally destroyed at the WTC. However, their conclusion, that we report below, seems not really convincing:

> "*The modeling implies that the contribution from the aircraft alone would yield the HTO deposition fraction of 2.5%. This value is too high by a comparison with other incidents involving fire and tritium. Therefore, the source term from the airplanes alone is too small to explain the measured concentrations, and another missing source is needed.*
>
> *There is evidence that weapons belonging to federal and law-enforcement agencies were present and destroyed at the WTC. Such weapons contain tritium sights by design. The exact activity of tritium from the weapons was not determined. The data and modeling are consistent with the tritium source from the weapon sights (plus possibly tritium watches) in the debris, from which tritium was slowly released in the lingering fires, followed by an oxidation*

and removal with the water flow. Our modeling suggests that such a scenario would require a minimum of 120 equipped weapons destroyed and a quantitative capturing of tritium, which is too high, since many weapons were found with only minor damage and tritium sights are shielded in a metal. Therefore, such a mechanism alone is not sufficient to account for the measured HTO concentrations. This indicates that the weapons/watches are consistent with the missing source, which would have complemented the airplane source."

According to us, this does not sound like an objective, scientific explanation of a phenomenon but rather like an attempt to make data stick to a preconceived theory. We would suggest to investigate the possibility of a nuclear explosion source, and not to limit this investigation to tritium but to all fission or fusion byproducts.

5.9 Cancer epidemy among first responders

Sadly, it appears that a very large number of first responders and workers that were breathing Ground Zero atmosphere during the weeks following the attacks developped cancers. According to an August 2018 New York Post article [64], "nearly 10 000 people have gotten cancer from toxic 9/11 dust".

As we have no medical competence, we will not elaborate on this but suggest that a nuclear origin of the diseases should also be investigated, as there are many radioinduced cancers.

However, it should be noted also that an underground nuclear explosion does not produce an extremely large amount of radioactive fallout, especially for contained explosions. As we already pointed out in a footnote on subsection 4.2.1, Teller *et al.* wrote in their 1968 book [30] (p. 3), suggesting to use fusion rather than fission devices to minimize health problems:

> "*The fusion explosions, however, can be handled in such a way as to eliminate most of the ensuing residual activity.*"

5.10 Visible cavity when cleaning Ground Zero

Although the idea of a nuclear destruction of the World Trade Center has not, to our knowledge, been reported in the scientific literature so far, it is not new in itself and has been claimed for at least ten years by number of individuals all over the Internet. Some consider that miniaturized nuclear explosives were planted in the buildings, which is false as we have shown, and others do claim that the explosives were planted deep underground, like we do. However they very often consider, among the proofs that lead to such a conclusion, the fact that strange cavities were found at Ground Zero during the cleaning process and before new building constructions, and they claim these cavities were indeed the initial cavities left by the underground explosives, which contradicts reports made by newspapers like the New York Times [65], or more specialized academic papers [66].

We claim here that there is no chance these cavities could be the ones left by the underground nuclear explosives, and that on the contrary, in order to admit the idea that three buildings were demolished using underground nuclear explosives at the World Trade Center, one must understand that it was necessary for the cavities to be produced deep underground and never to open onto the ground surface or even close to it. Such cavities had necessarily to be filled with rubble after the collapse of the cavity roof and subsequent formation of a rubble chimney, as exposed earlier. Only because of

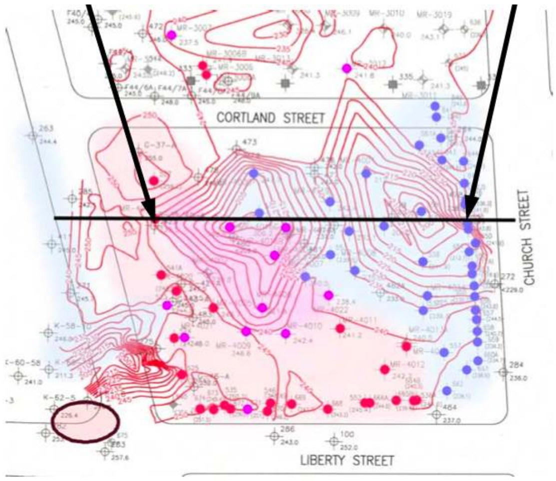

Figure 22: Location of 2 large depressions found at Ground Zero, according to Moss and Merguerian [66](Fig. 8 p. 8). Black line and arrows indicate the location of a cross-section not shown here.

this could a nuclear demolition device be designed in a "normal" - *i.e.* non-criminal - way, since radioactive contamination had to be confined as much as possible; although Teller *et al.* [30] do not specifically address the case of nuclear explosives used for high-rise building demolition, it can be logically induced from their book that in the mind of some engineers at this time - the 1960's - it was not a taboo and could even be seen as an economical and efficient way to achieve such a complex work.

Furthermore, we would like to point out the obvious fact that if the plunge pools and potholes found at Ground Zero were remnants of previously achieved demolitions, they would necessarily be located under the three buildings that collapsed on September 11, 2001. This was not the case, as can be found in Moss and Merguerian paper [66]; they were found beneath the Tower 4 site, *i.e.* a place where was previoulsly standing a small 9-story office building, WTC4, and where it was necessary to dig deeper in order to secure the foundations of a new and much higher (297 m) building, directly onto the bedrock. There is consequently no mistery in the fact that these large depressions remained unnoticed during the first construction of the World Trade Center. There is also no need to suspect some geologists to hide the truth and to be part of a plot against American citizens.

6 Concluding remarks

6.1 The slippery slope of higher mathematics and computer simulations refinements

We have shown in this paper that physics, even at its basic level, can be an extremely powerful tool for solving complex problems, provided it is used bearing in mind that no computer simulation, no fancy mathematics gan give a human mind a better undersanding than a rigorous and simple application of its fundamental laws expressed in natural, human language - such as, in our case, mainly first and second laws of thermodynamics, and the fundamental origins of chemical and nuclear energies.

We live in a time where mathematical tools and computing power - especially the latter - can be both extremely complex and powerful, which opens new research fields or technical applications that were some decades ago only to dream about. This is not to be regretted, just the opposite, but scientists must now face the idea that some of their activity has turned into something different from rational thinking, with high risks of producing a new kind of "magic thinking" or obscurantism. The fact that virtual reality is less and less discernable from real world should be a real concern since it tends to obscure understanding and make people - scientists or not - believe some events are real whereas they can be mere fabrications that violate very simple and thoroughly checked laws of physics, and that were planned by ill-intentioned individuals to fool others.

A similar danger comes from higher mathematics methods when they are used not because they are really necessary, but as a second-best solution when no simple explanation has been found, or worse, when the authors wants to hide some questionable aspects of their demonstration. If it is true that structural engineers need some refined mathematics methods - or powerful computer simulations - to get a precise solution of some complex problems, it is also true that even the most complicated calculations rely on simple physics laws, like Newton's laws of motion, not the other way round. And as a consequence must reflect the cause, any obviously invalid conclusion drawn from cumbersome calculations - like violation of energy conservation - must be rejected in the first place, without any need to "check the equations". As our Ph. D. advisor put it, *"Never make a calculation if you don't know in advance what it gives."*. Although it can sound a bit rough, it is basically correct.

6.2 Coming back to "simple" physics as our glorious ancesters did

Recently, the much celebrated physicist Stephen Hawking passed away and it has been recalled that his life and scientific activity, much longer than what was anticipated because of his illness, had been only possible thanks to a huge level of technical assistance and computerized achievements, such as a speech machine he could control with tiny cheek muscle contractions. This is certainly true, but only part of the story. As French anthropologist of science and technology Hélène Mialet showed [67], Hawking managed to live and to be scientifically productive such a long time thanks to an extremely important human support - he was by no way "alone" - and also because he managed to formulate problems in a rather simple, visual form; Penrose-Carter diagrams were an especially useful tool for him, since he was unable to use his hands to write down - even with the help of a computer - complicated equations and to solve them. So even the "pure mind" Stephen Hawking had to find shortcuts in demonstrations, and to practice physics in a rather direct and intuitive way rather than by solving pages of equations.

6.3 Power of physics, which is not psychology nor politics, comes from its "coldness" and rigor

Richard P. Feynman was famous not only for his scientific contributions and excellent teaching skills, but also for his ability to be "politically incorrect" as we would say today. Being a physicist, he considered physics as the queen of sciences, which was maybe a personal view, but he was right when he pointed out that some demonstrations in other sciences do not fulfill the logical rigor criteria that a physics demonstration is supposed to have. He was therefore considered sometimes as contemptuous with respect to other fields like psychology, which he denied the label of "true science".

But he could probably give us a lesson today, as a whole scientific community, with a few exceptions - a notable one is a paper published in Europhysics News by Jones *et al.* in 2016 [7] - has done almost nothing to question the dominant narrative around the terrorist attacks of September 11, 2001, even though it was obviously false from numerous scientific aspects. How was this possible? It is in our opinion a very interesting question, but not a physics one, ant this is why we will let others try to answer it.

We will only give a hint which is by no way a proof: professional scientists love to solve problems and hate to stumble over a demonstration. They also do not want to be seen as obscurantists, and it is true that among the people that question the dominant narrative we can find a large number that have only a faint idea of what science is. Therefore, as irrational as the "official story" was, they preferred to consider it true - or, at best, to consider it false but to think investigating the question was not worthy of their skills - rather to be seen accepting discussion with "weird" people. This is a wrong way to do science, since it introduces some psychological criteria - does this subject really deserve my expertise? Will I look ridiculous if I accept to work with these people? - where only rigorous thinking and approved knowledge should prevail. For a physicist, conservation of energy is not optional.

Science is defined by its method, not by its subject. The unavoidable specialization of scientific research has crumbled the scientific community into numerous peoples that use different tools and can hardly speak a common language, but the very basis of their work - human language, which conditions logic - is the same. We hope this work will help scientists all around the world, and especially physicists, remind that the most valuable part of their work resides in a rigorous use of fundamental knowledge, and not in fancy calculations or computer simulations that remain obscure for most people - and maybe even for themselves.

7 Acknowledgments

It would be particularly unfair to end this article without telling who is at the origin of our investigation. As we already said, number of people, who do not pretend to be scientists for most of them, have proposed for years a nuclear demolition explanation for the World Trade Center disaster. However, to our knowledge, only one person originated the "deep underground nuclear demolition" scheme: a man named Dimitri Khalezov, who pretends to be a former soviet officer specialized in nuclear weapons. He has also written a huge "report" of more than 1000 pages[42] - he pretends to be a "witness", not a "conspiracy theorist" - that he has made available for download on a website [68].

We have read this report (July 2013 version), with the exception of its appendices. Although it contains some errors (regarding steel thickness in WTC1 and WTC2 columns, for instance, or the fact that it considers the "WTC potholes" as nuclear cavities), we consider it gives a rather correct explanation of the way 3 skyscrapers were demolished on September 11, 2001. On other aspects however, this report might be considered as utterly unscientific and we *do not* claim to agree with all its content.

But interestingly, our first appreciation of this work was that it looked entirely unbelievable, especially because Dimitri Khalezov gives the yield of the nuclear devices: 150 kt each. It was precisely this feeling that made us performing a back-of-the-envelope calculation based on energy conservation, and to find out that, at least at the order of magnitude level, he was right if one considers the first law of thermodynamics to be valid. This was for us a lesson: never call someone "stupid" before checking what he (or she) says.

Some other people, like a German citizen named Heinz Pommer, who has also a background in nuclear energy (as a graduate physicist), have resumed and developped the idea of underground nuclear devices as an attempt to fully explain the strange phenomena that occurred at the World Trade Center on September 11, 2001. Heinz Pommer has made a great deal of effort making his work public for a large audience, and maintains currently two websites, one in German and in English [69], and one in German, Russian and English [70]. We consider that these websites contain a lot of useful information, but also some speculative arguments that do not fulfill the criteria of scientific research, which should proceed by elimination rather than by accumulation.

However, we are extremely grateful to Heinz Pommer for stimulating discussions and we

[42] Its title is "9/11thology: The "third" truth about 9/11 or Defending the US Government, which has only the first two...".

do acknowledge that his attitude is at the same time open-minded, benevolent and brave. Were only a small fraction of professional physicists in the world be able of such a “Mut zur Wahrheit[43]” attitude, would the 9/11 mystery be entirely solved and would physics as a science gain enormous sympathy and respect.

Finally, we would like to thank deeply our “old” friend François Sebesi who, although not a scientist according to academic criteria, has produced a great deal of scientific discourse in order to draw his fellows - including ourselves - out of a big collective illusion.

[43]“Courage to truth”

Appendix A: estimating energy released by radiative transfer

We argue in this paper that free convection is the main contributor to heat transfer in the cooling process of so-called "Ground Zero" after the terrorist attacks of September 11, 2001. Actually, since our aim is to find a *lower estimate* of the total amount of heat released, and since every mechanism adds a positive contribution to the total process because of the second law of thermodynamics which forces heat to flow spontaneously from a hot zone towards a colder one, knowing which mechanism is dominant is of secondary importance, and taking the wrong one only leads to underestimate the total amount of heat.

However, it is worth considering separately radiative transfer, since the non-linearity of Stefan-Boltzmann law (which scales as T^4) can lead to an important contribution, at least in the first part of the decay when the surface temperature is very high. For the same reason, it is not possible to take as a model an equivalent area of ground with uniform temperature like we did for free convection, but we must estimate the heat transfer rate by integrating power over some surface temperature distribution. In the following, we will assume that temperature at a given time t is distributed according to a gaussian law. Using a polar coordinate system, and taking for the origin the maximum temperature point, we use for the temperature distribution the following:

$$T\left(r,t\right) = T_\infty + \Delta T\, e^{-\alpha r^2} \tag{68}$$

where T_∞ is the ambient temperature, far from the hot zones, ΔT (which is a function of time) is the maximum temperature difference with T_∞ at $r = 0$, and $\alpha = \frac{1}{2\sigma^2}$, σ being the standard deviation which is here a length measuring the spatial extension of the hot zones. We will avoid in the following to use the σ notation for standard deviation since the same letter is also the usual notation for Stefan-Boltzmann constant.

As can be found in any standard textbook, the net radiative transfer rate between a surface of area S at temperature T_S with emissivity ϵ, considered as a "grey body", and a surrounding medium with a far-field temperature limit T_∞ reads:

$$P_{rad} = \epsilon\, \sigma\, S \left(T_S^4 - T_\infty^4\right) \tag{69}$$

where $\sigma = \frac{2\pi^5 k_B^4}{15 h^3 c^2} \approx 5.67 \times 10^{-8}\ \mathrm{W\,m^{-2}\,K^{-4}}$ is the Stefan-Boltzmann constant.

It follows that the net radiative transfer rate for a hot surface with a radial temperature distribution $T\left(r,t\right)$ will be expressed as the following integral over the surface, in polar coordinates:

$$\begin{aligned} P_{rad}\left(t\right) &= \epsilon\, \sigma \int_0^{2\pi} d\theta \int_0^{+\infty} \left(T\left(r,t\right)^4 - T_\infty^4\right) r\, dr \\ &= 2\pi\, \epsilon\, \sigma \int_0^{+\infty} \left(T\left(r,t\right)^4 - T_\infty^4\right) r\, dr \end{aligned} \tag{70}$$

Let us calculate $T\left(r,t\right)^4$ according to Equ. 68:

$$\begin{aligned} T\left(r,t\right)^4 &= \left[T_\infty + \Delta T\, e^{-\alpha r^2}\right]^4 \\ &= T_\infty^4 + 4T_\infty^3 \Delta T\, e^{-\alpha r^2} + 6T_\infty^2 \left(\Delta T\, e^{-\alpha r^2}\right)^2 + 4T_\infty \left(\Delta T\, e^{-\alpha r^2}\right)^3 + \left(\Delta T\, e^{-\alpha r^2}\right)^4 \end{aligned}$$

Therefore the integrand in Equ. 70 reduces to:

$$\begin{aligned} I(r) &= \left[4T_\infty^3\,\Delta T\,e^{-\alpha r^2} + 6T_\infty^2\left(\Delta T\,e^{-\alpha r^2}\right)^2 + 4T_\infty\left(\Delta T\,e^{-\alpha r^2}\right)^3 + \left(\Delta T\,e^{-\alpha r^2}\right)^4\right] r \\ &= \left[4T_\infty^3\,\Delta T\,e^{-\alpha r^2} + 6T_\infty^2\,\Delta T^2\,e^{-2\alpha r^2} + 4T_\infty\,\Delta T^3\,e^{-3\alpha r^2} + \Delta T^4\,e^{-4\alpha r^2}\right] r \end{aligned} \tag{71}$$

and the net radiative transfer rate can be written as the sum of 4 integrals:

$$P_{rad}(t) = 2\pi\,\epsilon\,\sigma\left[I_1 + I_2 + I_3 + I_4\right]$$

where:

$$\begin{aligned} I_1 &= \int_0^\infty 4T_\infty^3\,\Delta T\,e^{-\alpha r^2}\,r\,dr \\ I_2 &= \int_0^\infty 6T_\infty^2\,\Delta T^2\,e^{-2\alpha r^2}\,r\,dr \\ I_3 &= \int_0^\infty 4T_\infty\,\Delta T^3\,e^{-3\alpha r^2}\,r\,dr \\ I_4 &= \int_0^\infty \Delta T^4\,e^{-4\alpha r^2}\,r\,dr \end{aligned}$$

Let us now calculate each of these sums, which is quite straightforward since $\int_0^\infty e^{-\beta r^2}\,r\,dr = \left[-\frac{1}{2\beta}e^{-\beta r^2}\right]_0^\infty = \frac{1}{2\beta}$:

$$\begin{aligned} I_1 &= 4T_\infty^3\,\Delta T\int_0^\infty e^{-\alpha r^2}\,r\,dr \\ &= \frac{2}{\alpha}T_\infty^3\,\Delta T \end{aligned}$$

$$\begin{aligned} I_2 &= 6T_\infty^2\,\Delta T^2\int_0^\infty e^{-2\alpha r^2}\,r\,dr \\ &= \frac{3}{2\alpha}T_\infty^2\,\Delta T^2 \end{aligned}$$

$$\begin{aligned} I_3 &= 4T_\infty\,\Delta T^3\int_0^\infty e^{-3\alpha r^2}\,r\,dr \\ &= \frac{2}{3\alpha}T_\infty\,\Delta T^3 \end{aligned}$$

$$\begin{aligned} I_4 &= \Delta T^4\int_0^\infty e^{-4\alpha r^2}\,r\,dr \\ &= \frac{1}{8\alpha}\Delta T^4 \end{aligned}$$

Finally we get for the net radiative transfer rate[44]:

$$\begin{aligned} P_{rad}(t) &= 2\pi\,\epsilon\,\sigma\left[I_1 + I_2 + I_3 + I_4\right] \\ &= 2\pi\,\epsilon\,\sigma\left[\frac{2}{\alpha}T_\infty^3\,\Delta T + \frac{3}{2\alpha}T_\infty^2\,\Delta T^2 + \frac{2}{3\alpha}T_\infty\,\Delta T^3 + \frac{1}{8\alpha}\Delta T^4\right] \end{aligned} \tag{72}$$

[44] Note that although we did not write it explicitly to keep equations readable, ΔT is a function of time whereas T_∞ is assumed to be a constant.

Now to calculate the total energy transferred by radiation during the cooling process, we need to assume some particular form for ΔT as a function of time. When only linear equations are involved for heat transfer we get an exponential decay; in the case of radiative transfer we have non-linear equations because Stefan-Boltzmann law scales as T^4. At the beginning of the cooling process, for high temperature differences, the relative contribution of radiative transfer will therefore be larger than at the end; the temperature decay will consequently be faster in the early times than for a purely exponential decay.

However, let us try a self-consistent calculation and suppose first that radiative transfer is a relatively small contributor compared to other mechanisms (free convection, forced convection, conduction): assuming this we can say that ΔT will be approximatively exponentially decaying with time:

$$\Delta T \approx \Delta T_0 \, e^{-\frac{t}{\tau}} \tag{73}$$

where ΔT_0 is the maximum temperature difference at $t = 0$ and τ the characteristic time of the exponential.

We can now calculate the energy released by radiative transfer:

$$\begin{aligned} E_{rad} &= \int_0^\infty P_{rad}(t)\, dt \\ &= E_{rad,1} + E_{rad,2} + E_{rad,3} + E_{rad,4} \end{aligned}$$

where:

$$\begin{aligned} E_{rad,1} &= \frac{4\pi\,\epsilon\,\sigma}{\alpha} T_\infty^3\, \Delta T_0 \int_0^\infty e^{-\frac{t}{\tau}}\, dt = \frac{4\pi\,\epsilon\,\sigma}{\alpha} T_\infty^3\, \Delta T_0\, \tau \\ E_{rad,2} &= \frac{3\pi\,\epsilon\,\sigma}{\alpha} T_\infty^2\, \Delta T_0^2 \int_0^\infty e^{-\frac{2t}{\tau}}\, dt = \frac{3\pi\,\epsilon\,\sigma}{2\alpha} T_\infty^2\, \Delta T_0^2\, \tau \\ E_{rad,3} &= \frac{4\pi\,\epsilon\,\sigma}{3\alpha} T_\infty\, \Delta T_0^3 \int_0^\infty e^{-\frac{3t}{\tau}}\, dt = \frac{4\pi\,\epsilon\,\sigma}{9\alpha} T_\infty\, \Delta T_0^3\, \tau \\ E_{rad,4} &= \frac{\pi\,\epsilon\,\sigma}{8\alpha} \Delta T_0^4 \int_0^\infty e^{-\frac{4t}{\tau}}\, dt = \frac{\pi\,\epsilon\,\sigma}{32\alpha} \Delta T_0^4\, \tau \end{aligned}$$

Putting everything together we finally get:

$$E_{rad} = \frac{\pi\,\epsilon\,\sigma\,\tau}{\alpha}\left[4T_\infty^3\,\Delta T_0 + \frac{3}{2}T_\infty^2\,\Delta T_0^2 + \frac{4}{9}T_\infty\,\Delta T_0^3 + \frac{1}{32}\Delta T_0^4\right] \tag{74}$$

This energy is to be compared with the heat released by free convection for the same surface temperature distribution. As stated before in Equ. 9, the convective heat flux $\dot{q}$ coming from a surface can be expressed through a convection heat transfer coefficient h in the following way:

$$\dot{q} = h\,(T_S - T_\infty)$$

where T_S and T_∞ are the surface and bulk fluid temperature, respectively. For our gaussian temperature distribution the total heat flux, or free convection heating power (let us call it P_{fc}) will be:

$$\begin{aligned} P_{fc} &= h \int_0^{2\pi} d\theta \int_0^{+\infty} \Delta T\, e^{-\alpha r^2} r\, dr \\ &= 2\pi\, h\, \Delta T \int_0^{+\infty} e^{-\alpha r^2} r\, dr \end{aligned}$$

$$= 2\pi\, h\, \Delta T \left[-\frac{1}{2\alpha} e^{-\alpha r^2} \right]_0^{\infty}$$

$$= \frac{\pi\, h\, \Delta T}{\alpha} \tag{75}$$

And finally, integrating over time we get the total amount of heat Q_{fc} released by free convection during the cooling process:

$$\begin{aligned} Q_{fc} &= \int_0^{+\infty} P_{fc}\, dt \\ &= \frac{\pi\, h}{\alpha} \int_0^{+\infty} \Delta T_0\, e^{-\frac{t}{\tau}}\, dt \\ &= \frac{\pi\, h\, \tau}{\alpha} \Delta T_0 \end{aligned} \tag{76}$$

We can now compare the two values from Equ. 74 and 76. Let us call R_E the ratio between the energy released by radiative transfer and the heat released by free convection:

$$\begin{aligned} R_E &= \frac{E_{rad}}{Q_{fc}} \\ &= \frac{\epsilon\, \sigma}{h} \frac{\left[4T_\infty^3\, \Delta T_0 + \frac{3}{2} T_\infty^2\, \Delta T_0^2 + \frac{4}{9} T_\infty\, \Delta T_0^3 + \frac{1}{32} \Delta T_0^4\right]}{\Delta T_0} \end{aligned} \tag{77}$$

To estimate the relative importance of radiative transfer compared to free convection we must now choose some numerical values for ϵ, h, T_∞ and T_0. As proposed earlier (Equ. 19)[45] we retain $h \approx 10\,\mathrm{W.m^{-2}.K^{-1}}$and $T_\infty \approx 300\,K$. Emissivity ϵ of a surface can vary from very low values for highly polished metals ($\epsilon = 0.02$ for polished silver[46] at 300 K according to [15]) to values close to 1 for rough, non-conducting surfaces ($\epsilon \approx 0.9$ for concrete[47] and several building materials at 300 K according to the same source). Note however that emissivity can vary with temperature and is lower (as low as 0.4 for $T \approx 1000$ K) for some refractory materials at high temperatures according to several sources. We choose therefore $\epsilon \approx 0.8$ although we could as well put $\epsilon \approx 1$ for this rough estimate calculation. The maximum temperature difference (both in time and in space) ΔT_0 must be different from the 350 K value taken in Equ. 21 since this one was an equivalent value for a uniform temperature hot zone, a simplification which was possible because of the linearity of convective heat transfer equations. But as we showed from several sources in subsection 3.4, 5 days after the terrorist attacks some temperatures as high as 1000 K could be measured by IR radiation spectral analysis. We consequently take $\Delta T_0 \approx 1000 - 300 = 700$ K.

Let us now perform a numerical calculation:

$$\begin{aligned} R_E &\approx \frac{0.8 \times 5.67 \times 10^{-8}}{10} \frac{\left[4 \times (300)^3 \times 700 + \frac{3}{2} (300)^2 \times (700)^2 + \frac{4}{9} 300 \times (700)^3 + \frac{1}{32} (700)^4\right]}{700} \\ &\approx 4.54 \times 10^{-9} \frac{75.6 \times 10^9 + 66.15 \times 10^9 + 45.73 \times 10^9 + 7.50 \times 10^9}{700} \\ &\approx 4.54 \times 10^{-9} \frac{195 \times 10^9}{700} \\ &\approx 1.3 \end{aligned} \tag{78}$$

[45] We used then the slightly different notation $\overline{h}$ for an *average* heat transfer coefficient.

[46] hemispherical emissivity

[47] hemispherical emissivity

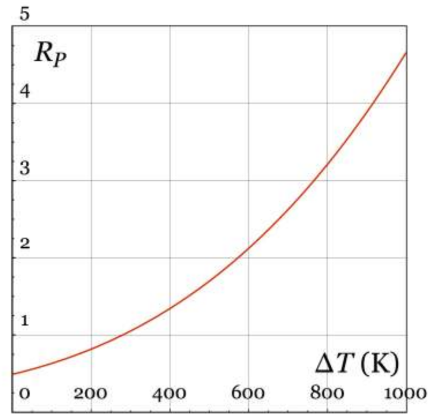

Figure 23: Variation of R_P as a function of ΔT. The larger the temperature difference, the more important becomes radiative transfer compared to free convection.

Given all the uncertainties of the problem[48], all we can conclude is that radiative and free convection contributions to the whole cooling process are about the same magnitude, which is contradictory to the hypothesis we have made earlier for imposing an exponential decay to temperature difference in Equ. 73.

However, as stated earlier, our purpose is to get an order of magnitude estimate of heat released at Ground Zero, using as simple calculations as possible. All mechanisms giving a positive contribution, we can only underestimate the sum if we choose to estimate the contribution of a mechanism which is not the dominant one. Furthermore, because free convection heat transfer equations are linear, calculation is much less sensitive to data errors than for radiative transfer and it may be safer to consider free convection mechanism instead of radiative transfer to get a total heat estimate, even if it is not the dominant mechanism, especially because we only need a lower boundary for this estimate.

Note that it is also possible to express a power ratio R_P instead of an energy ratio, giving the relative importance of radiative transfer compared to free convection for a given temperature difference at any time of the cooling process, irrespective of the temporal evolution of the temperature difference. Dividing Equ. 72 by Equ. 75 we get:

$$R_P = \frac{2\,\epsilon\,\sigma}{h}\left[2T_\infty^3 + \frac{3}{2}T_\infty^2\,\Delta T + \frac{2}{3}T_\infty\,\Delta T^2 + \frac{1}{8}\Delta T^3\right] \tag{79}$$

which we can plot as a function of ΔT to make this result more explicit. Using the same values as above for ϵ, h and T_∞ we get the graph in Fig. 23.

[48] And especially because, although "hot spots" at $T \approx 1000$ K have really been measured 5 days after the attacks, we can not ascertain that such high temperatures were distributed across the surface in a way that makes a gaussian distribution realistic.

Appendix B: The 1960s as the golden era of nuclear energy - a little reminder

We would like here to recall how common and "fashionable" was the use of nuclear energy in the 1960s - that is, when the World Trade Center was erected - and even for the use of nuclear explosives. This appendix is not strictly speaking of scientific nature, but rather is an attempt to prove that the solution we found for our problem[49] was not as "unthinkable" for engineers without any criminal intents as it may seem today, and that it looks even rather "rational" once put into historical context. Although we claim that our proof is a real one in the physics sense, and therefore does not need any other argument to be accepted, we think it is important for such a sensitive subject to make things as clear and precise as possible. Except in Orwell's dystopia, ignorance is never a strength.

As said before in subsection 4.1, some large programs about non-military use of nuclear explosives have been conducted both in the USA and in the USSR during the 1960s and the 1970s - and even up to the 1980s in the case of USSR. These programs were absolutely not secret, but just the opposite widely presented in mainstream media as great achievements of science and technology that could allow to carry out large projects, especially within the field of civil engineering, at a fraction of the cost needed with conventional methods. We will below give two examples of such mainstream publications in French magazines, *Sciences & Avenir*[50] (in 1965) and *Science & Vie*[51] (in 1958), two well-known popular science magazines.

Between October 29, 1956 and November 7,

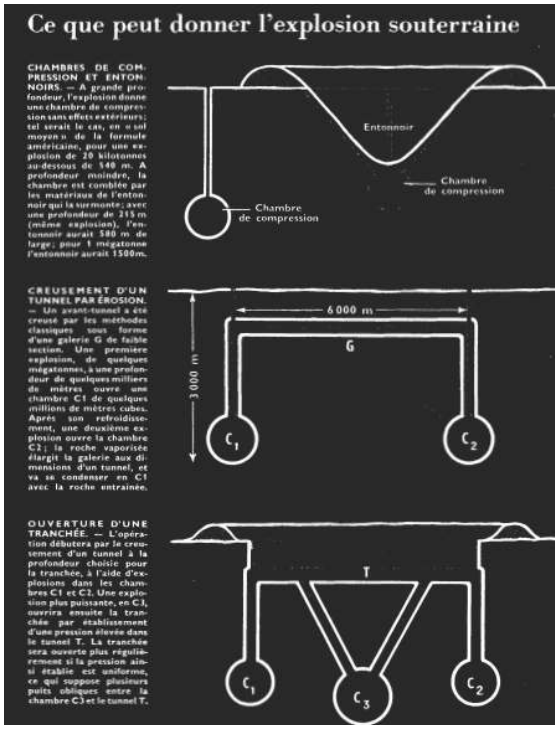

Figure 24: In 1958, French popular science magazine *Science & Vie* presents several civil engineering techniques using underground nuclear explosives. Depending on the depth of burial, different effects can be achieved.

[49] A solution that, let us recall it here, is the *only* possible one given the physical limitations of energy carriers.

[50] which translates into *Sciences and Future*

[51] which translates into *Science and Life*

1956 the so-called Suez Crisis gave number of engineers and scientists a good opportunity to begin promoting nuclear explosives as peaceful tools for large civil engineering projects. This invasion of Egyptian Sinai by Israel, followed by the United Kingdom and France, just after Egyptian President Gamal Abdel Nasser nationalized the Suez Canal, ended in a withdrawal of the three invaders after some political pressure of both USA and USSR. The event showed that the availability of such a strategic route as the Suez Canal - especially as a conduit for the shipment of oil - could be questioned, since Nasser responded to the attack by sinking all 40 ships present in the canal, which remained completely blocked until early 1957. As a consequence, it was suggested that a second canal could be made not depending on Egypt's will, using Israel's territory. But to carry out such a project at reasonable cost and time, only nuclear explosives appeared to be a viable solution.

If the project itself was abandoned, the idea remained and gave birth to similar projects. In October 1958, *Science & Vie* published an article [71] where the project of a nuclear-made harbour in Alaska was presented: it was Project Chariot (also abandoned later). But the paper not only mentioned this project: it also gave numerous other examples of what would be possible thanks to nuclear explosives in a near future. Among them: closing Bering Strait in order to warm the climate of some northern territories and make agriculture possible[52], doubling the Canal des Deux Mers in France[53] by a second one for large ships, or creating a Saharian inland sea... nothing seemed impossible to nuclear explosives, which were not only seen as modern tools, but also as cheap ones. For instance, it was written that "*If the port being prepared to dig in Alaska will be accessible to vessels of 90 m draft, it is for the sake of economy, and because it would cost too much to dig its basins only at*

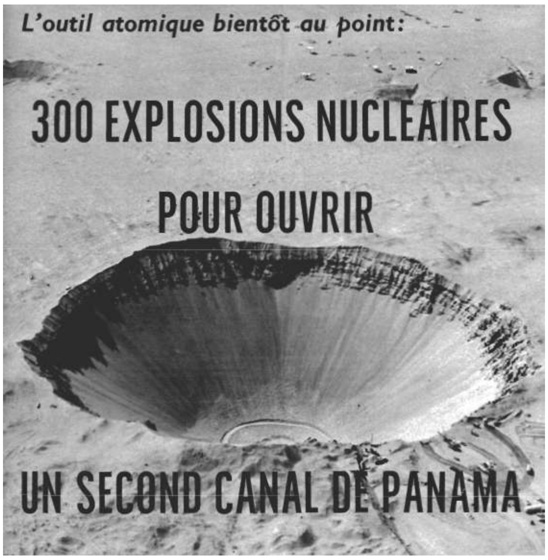

Figure 25: In a 1965 paper, French popular science magazine *Sciences & Avenir* presents an example of nuclear explosives use: "*300 nuclear explosions to open a second Panama Canal*". The title picture is the Sedan crater, created after the Sedan nuclear test in 1962, which was part of the *Operation Plowshare* program.

[52]Global warming was not, at that time, an issue.
[53]between Mediterranean Sea and Atlantic Ocean

15 m depth."[54]

In 1965 popular science magazine *Science & Avenir* also advocated the use of nuclear explosives for civil engineering in an article [72] about opening a second Panama Canal. The nuclear techniques were explained and although such explosions had obviously not to be confined underground, the issue of radioactive fallout was said to be controllable, provided thermonuclear explosives are used: "*It is obvious that only thermonuclear and not fission explosives should be used and that, in any case, it would be necessary to evacuate the population from relatively large areas.*"[55]. Anyway, the amount of energy required strongly pushed towards thermonuclear explosives: depending on the option chosen, a total between 170 and 270 Mt TNT equivalent should have been necessary, using explosives between 100 kt and 10 Mt each.

We will not argue further on this point and only advise readers to research the publications of this period in order to understand how much perception of nuclear energy has changed since, even in the case of nuclear explosives. We recall also that as we mentioned on a footnote in section 4.2, although it was made with a fission explosive, the soviet nuclear test Chagan in 1965, which resulted in an artificial lake, was declared safe by the authorities and that a short movie showed swimmers dipping into it shortly after, wearing only a small swimsuit.[50]

[54] "*Si le port qu'on s'apprête à creuser dans l'Alaska sera accessible aux navires de 90 m de tirant d'eau, c'est par raison d'économie, et parce qu'il coûterait trop cher de creuser ses bassins seulement à 15 m de profondeur.*"

[55] "*Il est évident qu'il ne faudrait employer que des explosifs thermonucléaires et non pas à fission et que, de toute façon, il serait nécessaire de procéder à l'évacuation de la population de zones assez étendues.*"

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